Reprint

Real-time FEXT Crosstalk Identification in ADSL Systems

N. Papandreou and Th. Antonakopoulos

The 2003 IEEE International Symposium on Intelligent Signal Processing – WISP 2003

BUDAPEST, HUNGARY, SEPTEMBER 2003

Real-time FEXT Crosstalk Identification in ADSL Systems

Nikolaos Papandreou[1] and Theodore Antonakopoulos[2]
[1]Academic Research Computer Technology Institute - CTI
61 Riga Feraiou Str., 26100 Patras, Greece
[2]Department of Electrical Engineering and Computers Technology
University of Patras, Rio, 26500 Patras, Greece

Abstract *– FEXT crosstalk identification results to improved frequency spectrum utilization in ADSL systems. The accurate determination of the crosstalk transfer function is a demanding task that is affected by various system impairments. In this paper we investigate the performance of a real-time method that exploits the exchange of signaling information of a new activated ADSL line in order to determine the crosstalk function between this line and an existing operational line.*

Keywords *– Crosstalk, Noise, Subscriber loops, Transfer functions*

I. INTRODUCTION

Digital subscriber lines (xDSL) technology provides high-speed data communication services to end-users utilizing the existing twisted-pair infrastructure. The maximum data rate of a DSL link depends on the characteristics of the loop environment in terms of line attenuation and noise level. Crosstalk interference induced by adjacent lines is one of the largest noise impairments that reduce the performance of services supported by the same binder [1]. As the support of more DSL services is required, the need for controlling the total interference becomes a critical issue. Spectrum management of DSL technologies refers to guidelines that minimize the potential of crosstalk interference and maximize the frequency spectrum utilization in multi-pair loop cables [2]. However as the demand for higher speed services increases and the number of DSL users continuous to grow, the development of methods for achieving coordination among various DSL modems in order to improve the total binder performance has become an appealing challenge. Dynamic spectrum management aims to the development of methods for transmission line and crosstalk identification that enhance the spectrum-management value [3], [4].

This work was partially supported by the "Karatheodoris" R&D program of the University of Patras and the Project 00BE33 entitled "Digital Subscriber Lines Technology" of the Greek Ministry of Industry.

Authors contact information: npapandr@cti.gr, antonako@ee.upatras.gr.

Recently, crosstalk identification in xDSL systems has attracted a lot of attention due to the significant benefits of having an accurate description of all cable services that generate crosstalk into a given pair. A non modem-based approach is presented in [5]. The crosstalk sources are identified in the frequency domain by finding the maximum correlation with a "basis set" of representative measured crosstalk coupling functions. The "basis-set" is generated by multiplying the canonical set of measured pair-to-pair crosstalk couplings with the power spectral density (PSD) of each specific type of DSL. However the concept of the "basis set" applies only to NEXT crosstalk where the "basis set" depends only on the disturber technology.

The idea of an impartial third party that identifies the crosstalk coupling functions among the twisted pairs of a binder is described in [6]. The third-party site collects the transmitted and received signals from all modems in the binder during a given time span. Initially, a cross-correlation technique is applied, in order to estimate the timing differences between the signals from different providers in the same bundle, and then a least-square method is used for estimating the crosstalk coupling functions and for finer scaling of the timing-offset among different operators. The concept of the impartial third-party site is based on the fact that usually the modems of the same bundle belong to different service providers, and thus it involves minimum coordination.

In this paper, we propose a crosstalk identification method for ADSL systems operating in the same binder. The method estimates the crosstalk coupling functions in real-time, by exploiting the initialization procedure of a new activated modem. Section II presents the system model used for studying FEXT and describes the algorithm for extracting the required information in a disturbed line. The timing synchronization and the crosstalk identification method are analyzed in Section III. Finally, Section IV presents various experimental results and demonstrates how the crosstalk identification method is affected by various system parameters.

II. FEXT MODEL AND SIGNALING DETECTION

Frequency division multiplexing (FDM) is used in order to avoid self-NEXT crosstalk in DSL systems [7]. Figure 1 shows an indicative FEXT interference environment, based on two downstream ADSL links, i.e. from the central office (CO) to the customer premises equipment (CPE). Given that one of the lines is already operational, called the *primary line*, we aim to determine a crosstalk identification mechanism at its CPE receiver, when another line, called the *disturber line*, is activated. In this Figure, line 1 is the primary line, while line 2 is the disturber line.

The initially activated primary line experiences only AWGN. During signaling [8], the receiver at the CPE side estimates the transfer function and noise power in the downstream link. Based on these measurements, the receiver determines the signal-to-noise ratio (SNR) of each subchannel and calculates the bit and gain distributions according to a bit-loading (BL) optimization algorithm. The BL algorithm is used to maximize the system performance in terms of total data rate and/or power, under a bit-error-rate (BER) requirement of 10^{-7} and a specific system margin in each subchannel [7].

When the second line is activated, the downstream receiver experiences AWGN and also FEXT crosstalk noise induced by the first line. The total noise level is again estimated along with the loop transfer function. Since the crosstalk coupling functions are not known, the receiver treats the total noise as Gaussian in order to calculate the BL distributions.

According to Recommendation G.992.1 [8], the SNR estimation is based on channel measurements of a specific wideband pseudo-random signal (PRD), sent by the far-end transmitter at high power level. As a result, the training signals of line 2 will also generate FEXT crosstalk noise to the receiver of line 1. If this FEXT noise level results in SNR degradation greater than the system margin, then the receiver of the first line experiences BER increase in some of its subchannels. However since the receiver of the primary line also knows the characteristics of the PRD signal, it can detect the activation of another line and estimate the respective crosstalk coupling function. The crosstalk identification process constitutes of the following major steps:

1. Detection of a new ADSL training sequence.
2. Synchronization on the PRD sequence frame.
3. Estimation of the crosstalk coupling function.

The signal received by the CPE modem of the primary line is corrupted by noise that consists of AWGN and FEXT components. In error-free transmission conditions, this noise can be estimated using the differences between the input and the output at the decoder stage. In case of erroneous decoding, the noise estimation is affected by the decoding errors, however in this case, the increased BER may result to re-execution of the channel training at the primary line. In the rest of this paper, we consider the case of error-free decoding with specific BER conditions.

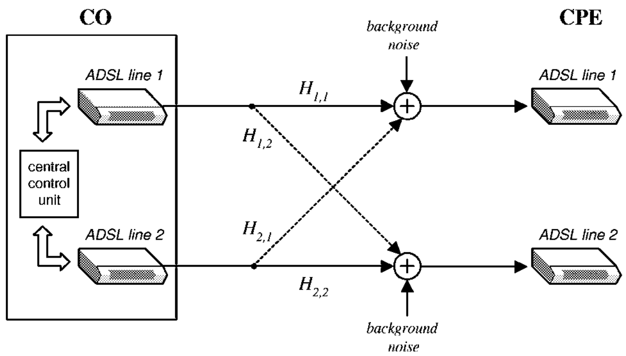

Fig. 1. FEXT crosstalk environment for FDM downstream ADSL transceivers.

A. Noise Frame Reconstruction

ADSL technology uses the discrete multi-tone (DMT) transmission scheme that decomposes the channel spectrum into a set of $N/2 = 256$ independent narrowband subchannels [9]. Figure 2 shows a general model of our primary DMT reception system. Modelling the channel as a finite impulse response (FIR) filter with L_{ch} real taps, the received time-domain sequence is given by:

$$y_k = h_k * x_k + u_k = \sum_{i=0}^{L_{ch}-1} h_i x_{k-i} + u_k \qquad (1)$$

where x_k is the transmitted signal, h_k is the channel impulse response and u_k represents the noise experienced at the receiver. In DMT systems, the data are transmitted using $N+\nu$ size blocks, where ν ($\nu \geq L_{ch} - 1$) is the size of the cyclic prefix that is added in order to avoid intersymbol interference (ISI). We choose $\nu = L_{ch} - 1$ in order to minimize the data overhead. Then denoting as x_k^r, $k = -\nu, \dots, -1, 0, \dots, N-1$ the samples transmitted at block r, where $x_{-i} = x_{N-i}$ for $i = 1, \dots, \nu$ is the cyclic prefix, we use (1) to form the following set of $N+\nu$ equations:

$$\left.\begin{array}{lllllll}
y_{-\nu}^r & = & h_0 x_{-\nu}^r & + & \dots & + & h_\nu x_{N-\nu}^{r-1} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
y_{-1}^r & = & h_0 x_{-1}^r & + & \dots & + & h_\nu x_{N-1}^{r-1} \\
y_0^r & = & h_0 x_0^r & + & \dots & + & h_\nu x_{-\nu}^r \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
y_{N-1}^r & = & h_0 x_{N-1}^r & + & \dots & + & h_\nu x_{N-\nu-1}^r
\end{array}\right\} \qquad (2)$$

The first ν equations determine the noiseless portion of the received block r, that is not used at the decoding process. These samples contain the ISI interference from the previous block $r-1$. The last N equations determine the noiseless useful samples that are processed by the receiver in order to extract the transmitted information. Using matrix notation and taking into account the noise, the useful portion of the received sequence can be written as:

$$\mathbf{y}_d^r = C\mathbf{x}_d^r + \mathbf{u}_d^r \qquad (3)$$

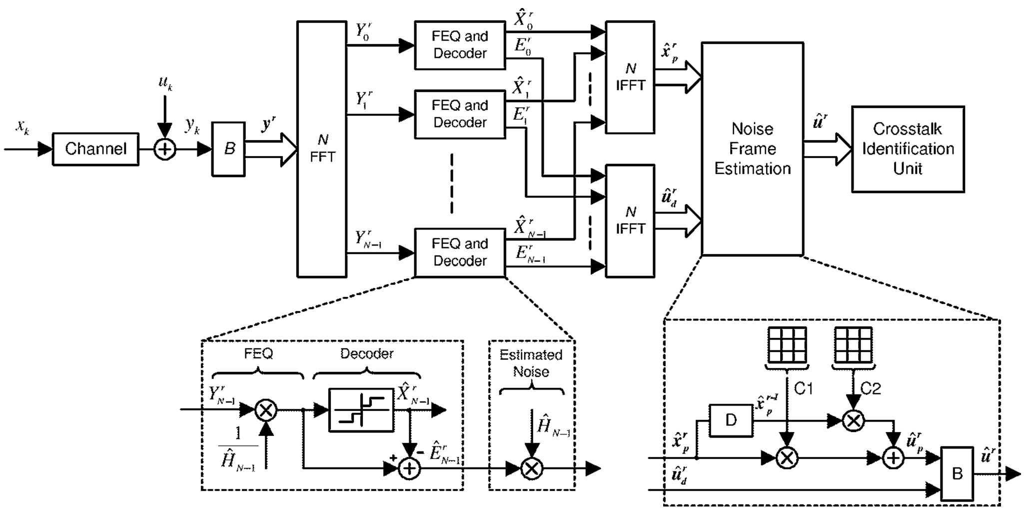

Fig. 2. General block diagram for noise frame estimation.

where $\mathbf{y}_d^r = [y_{N-1}^r, \dots, y_o^r]^T$, $\mathbf{x}_d^r = [x_{N-1}^r, \dots, x_o^r]^T$, $\mathbf{u}_d^r = [u_{N-1}^r, \dots, u_o^r]^T$, C is the channel response matrix and the subscript d is used to denote the useful data part. Given a cyclic prefix, the channel response matrix is circulant and can always be decomposed [10] as

$$C = Q^* \Lambda Q \tag{4}$$

where Q is the fast Fourier Transform matrix and Λ is a diagonal matrix whose diagonal elements correspond to the channel frequency response. The block $\mathbf{y}_d^r$ is provided to the receiver's FFT stage and the complex output $\mathbf{Y}_d^r$ is given by

$$\mathbf{Y}_d^r = Q\mathbf{y}_d^r = QC\mathbf{x}_d^r + Q\mathbf{u}_d^r = \Lambda \mathbf{X}_d^r + \mathbf{U}_d^r \tag{5}$$

where $\mathbf{X}_d^r = Q\mathbf{x}_d^r$ is the N-points FFT block of the useful transmitted samples and the noise $\mathbf{U}_d^r = Q\mathbf{u}_d^r$ has the same variance with $\mathbf{u}_d^r$. Note that the upper half of $\mathbf{X}_d^r$ corresponds to the transmitted subsysmbol sequence.

The signal is then passed to the frequency domain equalizer (FEQ), for attenuation and phase adjustment at each carrier, and then the slicer/decoder performs an estimation of the transmitted subsymbol sequence. A simple one-tap equalizer is used for multiplying the incoming subsymbol Y_n with the inverse estimated frequency response $\hat{H}_n$ of the corresponding subchannel, obtained during the transceiver training. We can define as Λ^{-1} the diagonal matrix, whose diagonal elements correspond to the inverse frequency response of the channel to represent the FEQ's operation on the FFT output. Assuming no decoding errors, $\hat{\mathbf{X}}_n = \mathbf{X}_n$, and perfect estimation of the primary channel, $\hat{H}_n = H_n$, we can calculate the error signal $\mathbf{E}_d^r$ between the input and the output of the decoder:

$$\mathbf{E}_d^r = \Lambda^{-1}\mathbf{Y}_d^r - \hat{\mathbf{X}}_d^r = \Lambda^{-1}\Lambda\mathbf{X}_d^r + \Lambda^{-1}\mathbf{U}_d^r - \mathbf{X}_d^r = \Lambda^{-1}\mathbf{U}_d^r \tag{6}$$

where $\Lambda^{-1}\Lambda = \mathbf{I}$ is the unitary matrix. Now if we further multiply $\mathbf{E}_d^r$ with Λ and perform IFFT, we can get an estimation of the time domain noise samples that correspond to the useful portion of the received block

$$\hat{\mathbf{u}}_d^r = Q^*\Lambda\mathbf{E}_d^r = Q^*\Lambda\Lambda^{-1}\mathbf{U}_d^r = \mathbf{u}_d^r \tag{7}$$

The process described previously can be used in order to estimate the noise samples u_k^r for $k = 0, \dots, N-1$ at block r. Now we are interested in estimating the noise samples for $k = -\nu, \dots, -1$.

Observing the first ν equations of set (2), we note that the received sequence depends on samples transmitted during the current block, x_k^r for $k = -\nu, \dots - 1$, and on samples transmitted during the previous block, x_k^{r-1}, for $k = N - \nu, \dots, N-1$. In particular, both subsets correspond to the cyclic prefix parts added in the two blocks and can be obtained from the decoded subsymbols $\hat{\mathbf{X}}_d^r$ and $\hat{\mathbf{X}}_d^{r-1}$ using IFFT. Using the cyclic prefix definition and taking into account the noise, we can write the first ν equations of (2) as

$$\mathbf{y}_p^r = \begin{bmatrix} C1 & C2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_p^r \\ \mathbf{x}_p^{r-1} \end{bmatrix} + \mathbf{u}_p^r \tag{8}$$

where $\mathbf{y}_p^r = [y_{-1}^r, \dots, y_{-\nu}^r]^T$, $\mathbf{x}_p^r = [x_{N-1}^r, \dots, x_{N-\nu}^r]^T$, $\mathbf{x}_p^{r-1} = [x_{N-1}^{r-1}, \dots, x_{N-\nu}^{r-1}]^T$, $\mathbf{u}_p^r = [u_{-1}^r, \dots, u_{-\nu}^r]^T$ and the subscript p is used to denote the cyclic prefix data part. The channel matrices $C1$ and $C2$ are defined as:

$$C1 = \begin{bmatrix} h_0 & h_1 & \dots & h_{\nu-1} \\ 0 & h_0 & \dots & h_{\nu-2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & h_0 \end{bmatrix} \tag{9}$$

$$C2 = \begin{bmatrix} h_\nu & 0 & \dots & 0 \\ h_{\nu-1} & h_\nu & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_1 & h_2 & \dots & h_\nu \end{bmatrix} \tag{10}$$

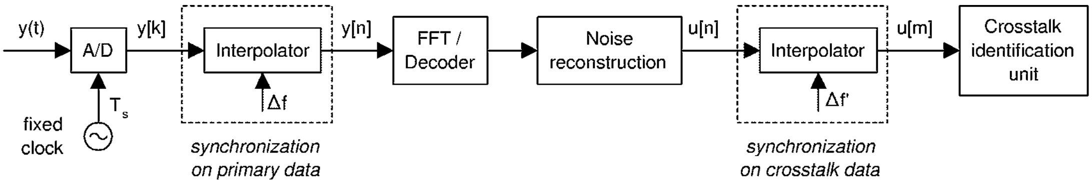

Fig. 3. General block diagram of synchronization units.

From (8) we can get an estimation of the noise samples u_k^r for $k = -\nu, \ldots, -1$:

$$\hat{\mathbf{u}}_p^r = \mathbf{y}_p^r - \begin{bmatrix} C1 & C2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_p^r \\ \mathbf{x}_p^{r-1} \end{bmatrix} = \mathbf{u}_p^r \tag{11}$$

The concatenation of the noise values, determined in equations (7) and (11), provides a complete estimate of noise at block r

$$\hat{\mathbf{u}}^r = [\hat{\mathbf{u}}_d^r \; \hat{\mathbf{u}}_p^r]^T \tag{12}$$

This noise signal contains AWGN and FEXT components, with the latter being the useful signal for the crosstalk identification process. It is obvious that due to the nature of the crosstalk coupling function, this signal has very low SNR.

B. ADSL Training Sequence Detection

Detection of the disturber's training sequence embedded in (12) is similar to the detection of types of signals transmitted during the ADSL initialization process [8]. In particular, initial detection can be obtained using a narrow bandpass filter in order to capture the transmission of pilot tones sent at specific subchannels. A second detection stage can be performed by a matched filter used to capture the transmission of the specific periodic patterns defined for ADSL systems. Both pilot tones and periodic patterns are transmitted over a large number of symbols during initialization and early enough before the transmission of the PRD for channel identification.

III. SYNCHRONIZATION AND CROSSTALK IDENTIFICATION

In DMT transmission systems we distinguish two types of synchronization: *sample synchronization* and *symbol synchronization* [11]. The first type guarantees frequency alignment between the receiver's and transmitter's sampling clocks, while the second type determines, from the received sample sequence, the boundaries of each DMT symbol, i.e. the $N + \nu$ samples that belong to the same symbol.

In our system model, we define as f_s the sampling frequency of the transmitter in line 1 and assume perfect synchronization with the far-end receiver. If we define a sampling frequency difference between the transmitter of the disturber line and the receiver of the primary line, this difference is embedded in the noise signal of (12). A second timing recovery unit is now responsible for synchronization on the crosstalk data stream. An all-digital timing correction scheme is presented in [12]. Figure 3 shows a general block diagram of the receiver of the primary line where the two different synchronization stages are presented.

The analysis of the interpolator's structure and the synchronization method used at the crosstalk identification unit is out of the scope of this paper. In order to include the effect of the timing recovery process, a residual frequency error, $\Delta f' = \hat{f}_s - f_s$, is assumed at the recovered noise signal and a fractional error $\epsilon = \Delta f' / f_s$ is defined. For ADSL systems the error ϵ is expected to be no more than ± 100 ppm (10^{-4}) [13]. The performance of the crosstalk identification method is analyzed in Section IV for different values of the frequency fractional error ϵ.

Regrading symbol synchronization, a similar approach is used as in the normal channel training sequence. Symbol synchronization exploits the periodicity of the incoming data frame, in order to estimate its boundaries. When a periodic sequence is received, its periodicity is determined by the length of the sequence. In the case of non-periodic DMT frames, either known or random, the periodicity is embedded in the cyclic prefix added in the useful data. In [14] a periodicity metric based on correlation of the incoming data with a delayed version is presented. During the line training procedures, a periodic pattern, named *Reverb*, is transmitted for a large number of symbols before starting the transmission of the PRD for channel estimation. In our model we assume frame synchronization on the recovered noise signal so that the receiver is able to identify the start of the PRD sequence.

The crosstalk function is identified using a least-square (LS) estimator. We model the crosstalk coupling function as an FIR filter of size L_{cr} and let K denote the number of samples used for estimation. The solution to the LS problem is given [15] by

$$\hat{\mathbf{h}} = (T^* T)^{-1} T^* \hat{\mathbf{u}} \tag{13}$$

where $\hat{\mathbf{h}}$ is the unbiased estimation of the crosstalk impulse response vector of size L_{cr}, $\hat{\mathbf{u}}$ is the input noise vector of size K and T is the Toeplitz matrix of the known PRD sequence with size $K \times L_{cr}$. Note that the noise vector $\hat{\mathbf{u}}$ is generated over a duration of K samples, as described in (12). Let $\mathbf{h} = [h_0, h_1, \ldots, h_{L_{cr}-1}]^T$ and $\hat{\mathbf{h}} = [\hat{h}_0, \hat{h}_1, \ldots, \hat{h}_{L_{cr}-1}]^T$ denote the actual and the estimated crosstalk vectors. In order to evaluate the estimation accuracy we use the following metric:

$$M = \frac{||\mathbf{h}||^2}{E[||\Delta \mathbf{h}||^2]} \tag{14}$$

where $\Delta\mathbf{h} = \hat{\mathbf{h}} - \mathbf{h}$ is the estimation error vector and $||\cdot||$ denotes the norm of the vector. The above metric represents the signal-to-estimation error ratio.

IV. SIMULATION RESULTS

In this section we present simulation results that demonstrate the estimation accuracy of the crosstalk transfer function based on the method described previously. The simulation model is based on the system depicted in Figure 1. The subscriber loops are considered to be 26 AWG 9 kft and correspond to the standard #6 CSA test-loop [8]. The crosstalk transfer function used in the system is modelled according to [2]. The estimation is examined under various system parameters including the synchronization error ϵ defined in the previous section, the number of samples K used for the LS method and the AWGN level presented in the system.

Figure 4a shows the estimated transfer function relative to the ideal crosstalk transfer function, for -140 dBm/Hz AWGN and 10000 samples, under different synchronization errors ϵ. We observe that the higher the frequency fractional error, the greater is the variation, especially in subchannels with stronger attenuation. In fact, due to the synchronization error, the observed signal $\hat{\mathbf{u}}$ of the LS method corresponds to different time span of the known PRD signal, although the same number of samples is used for both signals. As a result the method produces an estimate that differs from the original transfer function as the ϵ increases. For $\epsilon > 10^{-4}$, the estimated transfer function exhibits significant variations. In Figure 4a we observe that for $\epsilon = 10^{-5}$ and for 10000 samples, a good approximate of the original crosstalk transfer function is achieved.

Figure 4b presents the transfer function estimation under different numbers of used samples, for -140 dBm/Hz AWGN and $\epsilon = 10^{-5}$. The estimation accuracy also depends on the number of samples used in the LS method. For 4000 samples the estimation exhibits large variation, while for 20000 samples the variation decreases significantly.

Figure 4c presents the estimated transfer function, under different levels of system noise, for $\epsilon = 10^{-5}$ and 8000 samples. First, we observe that the case of -140 dBm/Hz produces intermediate estimation results compared to the plots of 4b for 4000 and 10000 samples. The other two plots of Figure 4c correspond to 3 dB and 6 dB higher noise levels respectively. It is obvious that for a given number of samples, the variation increases with the noise level. In this case using more samples would improve the estimation accuracy. The plots of Figure 4 clearly demonstrate the effect on the crosstalk identification method of the system noise level, the synchronization error at the receiver and the number of samples used in the LS estimation method.

The effect of the system noise and synchronization error can be further examined using the metric of (14), which represents the SNR of the estimation process. Figure 5 shows this metric as a function of the frequency fractional error and for different

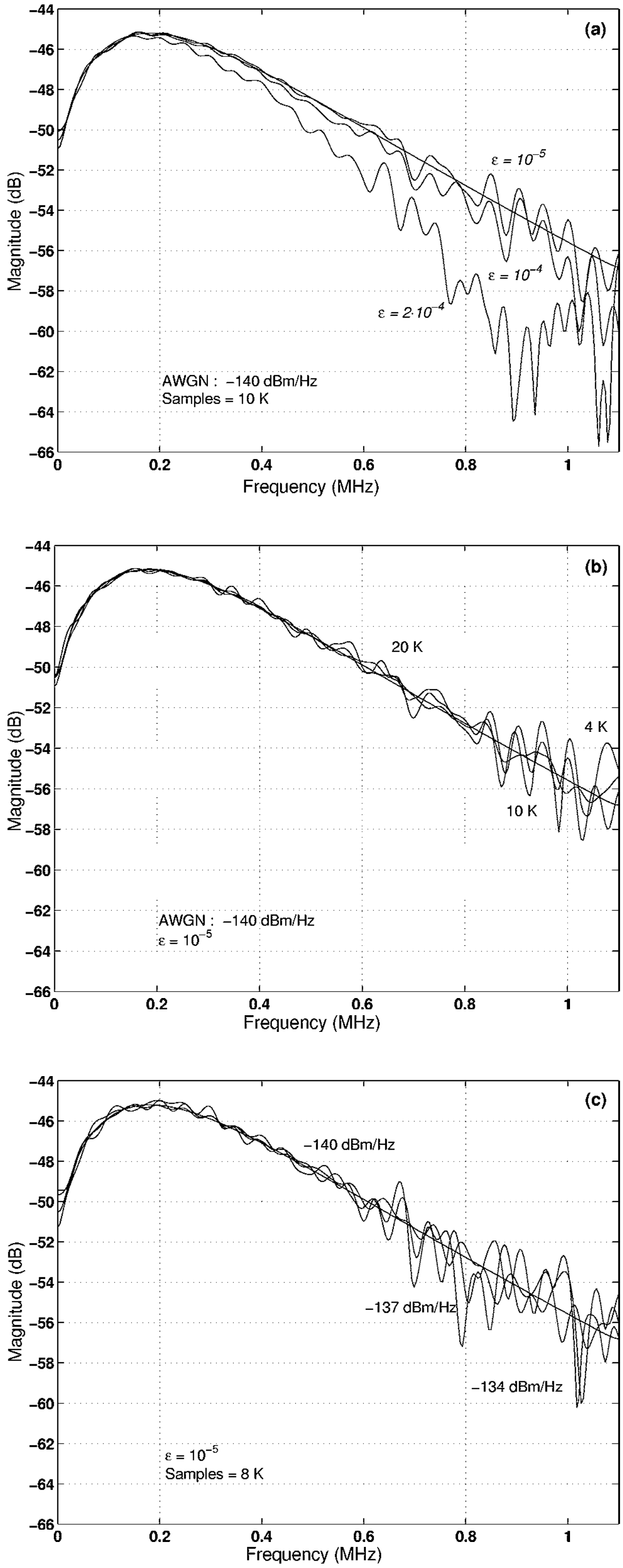

Fig. 4. The transfer function estimation dependence on: (a) the frequency fractional error, (b) the used samples and (c) the noise level.

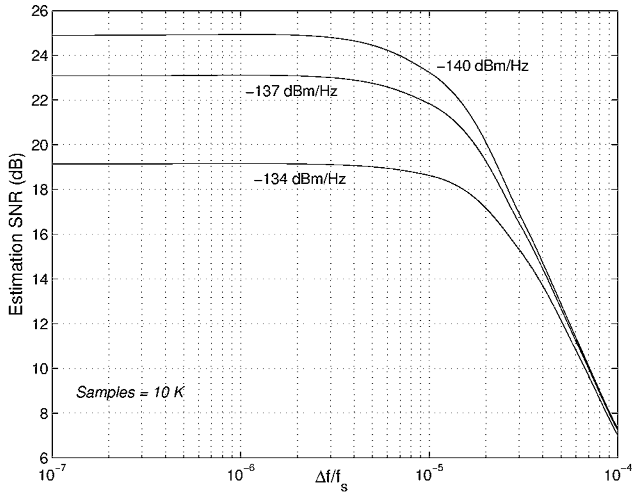

Fig. 5. Estimation SNR as a function of the frequency fractional error.

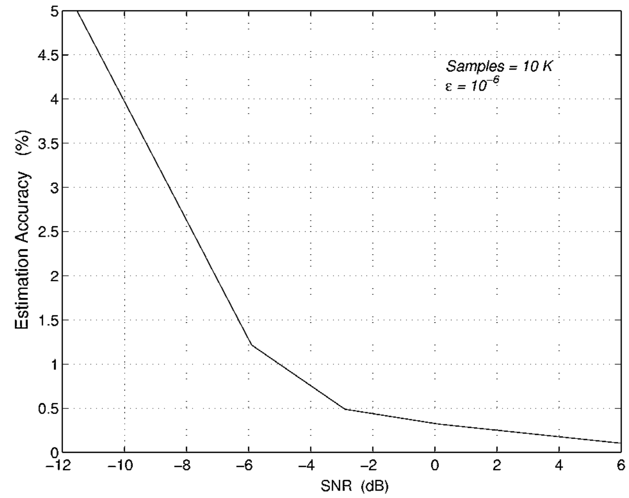

Fig. 6. Crosstalk identification accuracy as a function of SNR.

noise levels. The number of samples used for the estimation is 10000. We observe that for small values of the synchronization error, the metric remains constant and depends only on the noise level. When the synchronization error exceeds a specific value, the error introduced due to the frequency offset becomes dominant and the noise level has almost no effect. Small frequency fractional errors ϵ produce a quite similar output sequence at the second synchronization unit of Figure 3, so that the estimation error vector $\Delta\mathbf{h}$ changes slightly. Moreover, as also indicated in Figure 4c for constant number of samples, the higher the noise level, the less accurate is the approximation of the ideal crosstalk transfer function, and so the lower is the estimation SNR. When the synchronization error exceeds the value of 10^{-4}, no reliable results can be obtained using the LS estimation method.

As the method's estimation accuracy (in percentage) we define the inverse of the metric of (14). Figure 6 shows how the estimation accuracy depends on the SNR at the input of the crosstalk identification unit when the synchronization error remains low. The SNR is defined as the ratio of the power of the crosstalk injected signal to the power of AWGN in the primary line. This figure demonstrates that, for a given background noise in the primary line, the use of the LS method, in terms of the lower possible limit, depends on the magnitude of the crosstalk transfer function, even if no synchronization error is encountered.

V. CONCLUSIONS

This paper described a crosstalk identification method that is based on measurements performed during the transceiver training phase of ADSL modems operating in the same binder and demonstrated simulation results on a standard DSL test-loop. We examined the accuracy of the identification method regarding system parameters including noise level, synchronization error at the receiver and number of samples used in the estimation method. The presented "on-the-fly" estimate of the FEXT coupling function can be exploited by a central management unit located at the Central Office in order to optimize the bit and power distributions of all ADSL lines operating in the same binder.

REFERENCES

[1] J. W. Cook *et al.*, "The noise and crosstalk environment for ADSL and VDSL systems," *IEEE Commun. Mag.*, vol. 37, pp. 73–78, May 1999.

[2] "Spectrum Management for Loop Transmission Systems," ANSI Std. T1.417-2001, Jan. 2001.

[3] J. M. Cioffi *et al.*, "Proposed Scope and Mission for Dynamic Spectrum Management," T1E1.4/2001-188R4, Greensboro, NC, Nov. 2001.

[4] Kee Bong Song *et al.*, "Dynamic Spectrum Management for Next-Generation DSL Systems," *IEEE Commun. Mag.*, vol. 40, pp. 101–109, Oct. 2002.

[5] S. Galli, C. Valenti, and K. J. Kerpez, "A Frequency-Domain Approach to Crosstalk Identification in xDSL Systems," *IEEE J. Select. Areas Commun.*, vol. 19, pp. 1497–1506, Aug. 2001.

[6] C. Zheng *et al.*, "Crosstalk Identification in xDSL Systems," *IEEE J. Select. Areas Commun.*, vol. 19, pp. 1488–1496, Aug. 2001.

[7] T. Starr, J. M. Cioffi, and P. J. Silverman, Eds., *Understanding Digital Subscriber Line Technology*, Upper Saddle River, NJ: Prentice-Hall, 1999.

[8] "Asymmetrical Digital Subscriber Line (ADSL) Transceivers," ITU-T Recommendation G.992.1, July 1999.

[9] J. M. Cioffi, "A Multicarrier Primer," ANSI Contribution T1E1.4/91-157, Clearfield, Florida, Nov. 1991.

[10] C. Zeng and J. M. Cioffi, "Near-End Crosstalk Mitigation in ADSL Systems," *IEEE J. Select. Areas Commun.*, pp. 1497–1506, Aug. 2001.

[11] T. Pollet and M. Peeters, "Synchronization with DMT Modulation," *IEEE Commun. Mag.*, pp. 80–86, Apr. 1999.

[12] E. Martos-Naya *et al.*, "Optimized Interpolator Filters for Timing Error Correction in DMT Systems for xDSL Applications," *IEEE J. Select. Areas Commun.*, vol. 19, pp. 2477–2485, Dec. 2001.

[13] N. P. Sands and K. S. Jacobsen, "Pilotless Timing Recovery for Baseband Multicarrier Modulation," *IEEE J. Select. Areas Commun.*, vol. 20, pp. 1047–1054, 2002.

[14] P. R. Chevillat, D. Maiwald, and G. Ungerboeck, "Rapid Training of a Voiceband Data-Modem Receiver Employing an Equalizer with Fractional-*T Spaced Coefficients*," *IEEE Trans. Commun.*, vol. 35, pp. 869–876, Sep. 1987.

[15] N. Benvenuto and G. Cherubini, Eds., *Algorithms for Communications Systems and their Applications*, Chichester: Wiley, 2002.