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Computer Communications

VOL. 24 NO. 18, DECEMBER 2001, pp. 1759-1768

Computer Communications 24 (2001) 1759–1768

computer communications

www.elsevier.com/locate/comcom

Equilibrium point analysis of the binary exponential backoff algorithm

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Received 5 June 2000; revised 7 February 2001; accepted 15 February 2001

Abstract

A Markovian model of the backoff algorithm used in the slotted 1-persistent carrier sense multiple access/collision detection (CSMA/CD) access protocol is developed in order to analyze the performance of such a network. Due to its complexity, the existing models of the CSMA/CD protocol do not incorporate the effects of the backoff algorithm. In this work, we developed an approximate Markovian model of the system with a multidimensional state vector and we used the equilibrium point analysis technique in order to analyze the model. The throughput–delay characteristics and the stability behavior of the system are extracted, which conform the simulation results and the predictions of other theoretical models presented in the literature. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Ethernet; Binary exponential backoff; Equilibrium point analysis

1. Introduction

The IEEE 802.3/Ethernet, the most widespread LAN technology, uses the 1-persistent carrier sense multiple access/collision detection (CSMA/CD) to control access of multiple stations to a single shared medium and uses the truncated binary exponential backoff algorithm for collision resolution. Several theoretical performance analysis works of the Ethernet network have been reported [1–3]. However, most of these works have focused on the effects of 1-persistent CSMA/CD without incorporating the effects of the backoff algorithm. A model that incorporates the effects of the backoff algorithm results in a complicated multidimensional Markov chain and cannot be solved using queuing theory. This problem can be solved using approximate techniques.

This paper analyzes the dynamic behavior and performance of the backoff algorithm in a slotted 1-persistent CSMA/CD system for the single buffer case. The Markovian model of this system consists of a Markov chain with a multidimensional state vector whose components have strong dependency on each other. In order to analyze this Markov chain, we utilize an approximate analytic technique called equilibrium point analysis (EPA) proposed by Fukuda and Tasaka [4,5]. The EPA can easily analyze multidimensional Markov chains since it is not necessary to calculate state transition probabilities: the EPA assumes that the system is always at an equilibrium point.

The performance of an Ethernet network is highly dependent on the binary exponential backoff algorithm. Even at high speeds, Ethernet networks have some limitations for supporting real time and multimedia traffic, especially due to the well-known capture effect. Several solutions to the capture effect have been proposed in the literature. Most of these studies have focused on improving network performance by changing the BEB algorithm [6–9]. However, since the backoff mechanism is implemented in hardware on every end-station, such a solution requires changes in all nodes of the network and is not considered feasible. For that reason, new technologies, like PACE, have been used in Fast Ethernet hubs. PACE technology overcomes the capture effect problem without the drawbacks of solutions based on the modifications of the binary exponential backoff algorithm. PACE requires that the changes are made on the hub only and does not require any modifications at the end-stations. PACE algorithm reduces the access delay and jitter as compared to standard Ethernet.

The mathematical model of a network employing PACE results in a complicated multidimensional Markov chain and cannot be solved using queuing theory. In order to analyze such a network, the EPA can be used since it is not necessary to calculate the Markov chain state transition probabilities. The analysis of the binary exponential backoff algorithm can be used as the reference for the analysis of PACE and other similar algorithms used in Fast Ethernet switches and hubs.

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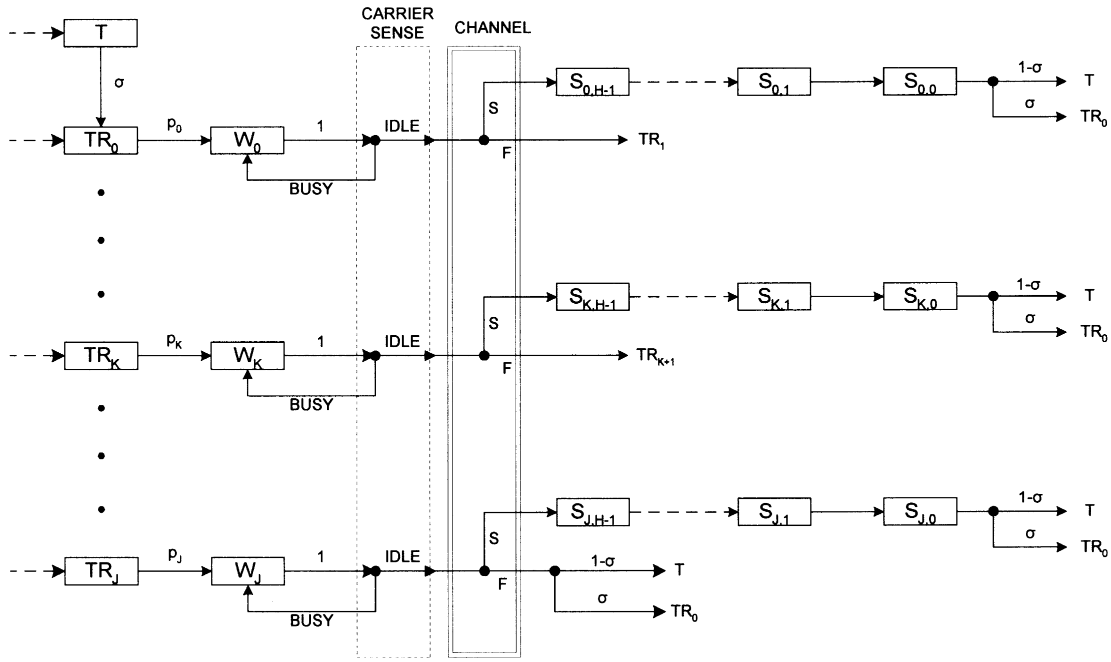

Fig. 1. An approximate model of the backoff algorithm in a CSMA/CD network.

The capture effect causes transient unfairness and substantial performance degradation to Ethernet networks, especially in two-station networks. The degree of transient unfairness is severe for small numbers of contending stations and it reduces quickly as the number of active stations increases. Table 2 in Ref. [9] shows that the channel holding time by a single user is roughly inversely proportional to the number of active users and the packet sizes. In an Ethernet network in which the users are connected through a hub, the number of users is not considered small. Therefore, the analysis presented in this paper can be applicable to network topologies (i.e. hub) that the capture effect is minimized due the large number of users.

The Markovian model of the system is described in Section 2. In Section 3, we derive the basic equations of EPA, which are applied in Section 4 in order to get the analytic formulas for the network throughput–delay analysis. Section 5 presents the numerical results from the above analysis and examines the network's performance.

2. The model analysis

The medium access control protocol in Ethernet networks consists of the following parts:

- the 1-persistent CSMA/CD algorithm that describes the rules a user must follow while trying to acquire the medium and
- the binary exponential backoff algorithm that determines the retransmission delay after an unsuccessful attempt.

In this paper, we deal with the slotted 1-persistent CSMA/CD protocol. In this version of CSMA/CD protocol, the time axis is considered slotted and the slot size is equal to the end-to-end propagation delay. All users are synchronized and forced to start transmission only at the beginning of a slot. A user senses the channel prior to transmission. If the channel is sensed idle, the user transmits the packet with probability one. If the channel is sensed busy, the user waits until the channel goes idle and only then transmits the packet. If a collision is detected, the user aborts transmission and then schedules its packet for retransmission after a random interval, computed by using the truncated binary exponential backoff algorithm. According to this algorithm, the mean retransmission interval is doubled in each of the first 10 successive collisions of the same packet. For the next attempts, the interval is truncated and remains at its last value. After 16 successive collisions the packet is rejected.

The following assumptions (as defined in Ref. [5]) are made regarding the characteristics of the users population and the operation of the backoff algorithm in a CSMA/CD network.

- The channel propagation delay is identical for all users and it is τ seconds; thus, the duration of a slot is τ seconds.
- The carrier sensing operation is performed instantaneously.

- The network consists of M users who communicate over a common broadcast channel.
- Each user generates a packet in a slot with probability σ. Each packet is of constant length requiring $H = 1/\alpha$ slots for transmission. Since the channel will be sensed idle, one slot after the end of a packet transmission, a successful transmission period is of length $(H + 1)$ slots.
- Each user has a single buffer capacity; thus, each user is not allowed to generate a new packet until the current packet has been successfully transmitted or permanently rejected.
- When the backoff period, computed according to the binary exponential backoff algorithm, has expired, a user senses the channel at the beginning of the next slot.
- When a collision occurs, all users seize their transmissions during the current slot; thus, the length of an unsuccessful transmission period is considered to be equal to one slot. Although in real CSMA/CD networks, the length of each unsuccessful transmission period is equal to the round-trip time (which is equal to two slots), the above assumption is considered since it simplifies the mathematical model, while it has minimum effect on the result's accuracy.
- No transmission errors occur.

Under the above assumptions, we can formulate an approximate model of the system shown in Fig. 1. At a given slot, each user is in one of the modes and might move into another at the next slot. We formulate the model by observing the state of the system at the beginning of each slot, assuming that the mode transitions occur at the end of each slot.

Initially, users that have no packet to transmit are in T mode. If a new packet arrives, the user moves to TR_0 mode. A user in TR_k mode had k unsuccessful attempts to transmit its packet and has a probability p_k to move into the W_k mode at the next slot, where $p_k = 1/2^k$ for $0 \le k \le 10$ and $p_k = 1/2^{10}$ for $11 \le k < 16$. A user in W_k mode senses the channel at the beginning of the current slot with probability one. When a user in W_k mode senses the channel idle, it immediately initiates the packet transmission. If the transmission is successful, then the user will move into $S_{k,H-1}$ mode at the next slot. If the transmission is unsuccessful, the user moves to the TR_{k+1} mode (for $0 \le k < J$). A user in W_J mode has J unsuccessful attempts to transmit, where $J = 15$; consequently, if the new transmission attempt is not successful, the user rejects the packet, and a new one can be accepted. Thus, a user in W_J mode will move into TR_0 mode with probability σ in case of new packet arrival, otherwise, the user will move to T mode with probability $1 - \sigma$. A user in S_{ki} mode (for $1 \le i \le H - 1$) has succeeded in transmitting a packet and will complete the transmission after i slots. Therefore, he will move into $S_{k,i-1}$ mode at the next slot with probability 1. A user in S_{k0} has just completed a successful transmission of a packet and is waiting for the packet to reach its destination. Thus, this mode demonstrates the influence of the channel propagation delay.

According to Ref. [5], let n_k be a random variable representing the number of users in TR_k mode, ν_κ the number of users in W_k mode, and m_{ki}, the number of users in S_{ki} mode. (Note that m_{ki} is either 1 or 0). Thus, the vector $n \triangleq (n_k, \nu_k, m_{ki} : 0 \le k \le J, 0 \le i \le H - 1)$ is a state vector of the system, which forms a discrete-time Markov chain with finite state space. The components of this state vector are strongly dependent on one another and the constraint $\sum_{i=0}^{H-1} \sum_{k=0}^{J} m_{ki} = 0$ or 1 is applied, which elaborates the fact that there can be only one successful transmission at a time.

In order to analyze the Markov chain, we utilize the EPA technique [4] due to the strong dependency among the components of the state vector and the large number of its possible states. The EPA is an approximate analytic tool that can easily analyze multidimensional Markov chains since it is not necessary to calculate state transition probabilities. The EPA assumes that the system is always at an equilibrium point [4,5]. By considering the fact that all nodes are in an equilibrium point, we can get a set of simultaneous equations whose solution gives one or more equilibrium points.

3. Equilibrium point analysis

In the following analysis, we assume that the system is in steady state. Given that the channel in state n is idle, a user in W_k mode can successfully transmit a packet only if there is no other user in W_k mode. Let $S_k(n)$ be the conditional probability that a user in W_k mode successfully transmits a packet, given that the channel in state n is idle. This probability depends on the number of users that have moved from the TR_k modes in the previous slot, as well as the channel state at the previous slot. Specifically, we have a successful transmission at the following cases:

- The channel was idle at the previous slot and only one user moved from TR_k to W_k at the previous slot
- There was a collision at the previous slot and only one user moved from TR_k to W_k at the previous slot
- The channel was busy at the previous slot and only one user moved from TR_k to W_k at the $(H + 1)$ transmission slots

Considering the fact that the duration of an unsuccessful transmission period is one slot, the second case can be regarded as part of the first. Then, the two terms that express the conditional probability $S_k(n)$ are

$$S_{k1}(n) = \left[n_k p_k (1 - p_k)^{n_k - 1} \prod_{i=0, i \ne k}^{J} (1 - p_i)^{n_i} \right] P_I, \quad (0 \le k \le J) \tag{1}$$

and

$$S_{k2}(n) = \left[n_k p_k (1-p_k)^{n_k-1} \prod_{i=0, i\neq k}^{J} (1-p_i)^{n_i} \left(\prod_{i=0}^{J} (1-p_i)^{n_i} \right)^H \right] \times (1-P_I), \quad (0 \leq k \leq J) \tag{2}$$

where

$$P_I(n) \triangleq \begin{cases} 1 & \text{if the channel at state } n \text{ is idle} \\ 0 & \text{if the channel at state } n \text{ is busy.} \end{cases} \tag{3}$$

We make the approximations of Ref. [5] that

$$(1-p_i)^{n_i} \simeq \mathrm{e}^{-n_i p_i} \text{ and } p_i/(1-p_i) \simeq p_i, \quad (0 \leq i \leq J) \tag{4}$$

Appendix A explain the validity of these approximations. By definition,

$$G_k(n) \triangleq n_k p_k \tag{5}$$

and

$$G(n) \triangleq \sum_{k=0}^{J} G_k(n) = \sum_{k=0}^{J} n_k p_k \tag{6}$$

Then, from Eqs. (1) and (2), the conditional probability $S_k(n)$ is expressed as

$$S_k(n) = G_k(n)\mathrm{e}^{-G(n)} P_I + G_k(n)\mathrm{e}^{-(H+1)G(n)}(1-P_I), \quad (0 \leq k \leq J) \tag{7}$$

For the rest of this work, we denote $S_k(n)$ as S_k, $G(n)$ as G, and $P_I(n)$ as P_I, except in cases where this notation is ambiguous.

After calculation of the conditional expectations of increase in the number of users in each mode of Fig. 1 in a slot, given that the system is in state n, and setting each of them equal to zero [5], we get the following equations:

T mode:

$$\left\{ M - \sum_{k=0}^{J} \left(n_k + \nu_k + \sum_{i=0}^{H-1} m_{ki} \right) \right\} \sigma = (1-\sigma) \sum_{k=0}^{J} m_{k0} + (1-\sigma)(\nu_J - S_J) P_I \tag{8}$$

W_k mode:

$$n_k p_k = \nu_k P_I \tag{9}$$

TR_k mode:

$$n_k p_k = (\nu_{k-1} - S_{k-1}) P_I, \quad (1 \leq k \leq J) \tag{10}$$

TR_0 mode:

$$n_0 p_0 = \sigma(\nu_J - S_J) P_I + \sigma \sum_{k=0}^{J} m_{k0} + \left\{ M - \sum_{k=0}^{J} \left(n_k + \nu_k + \sum_{i=0}^{H-1} m_{ki} \right) \right\} \sigma \tag{11}$$

S_{k0} through $S_{k,H-1}$ modes:

$$m_{k0} = m_{k1} = \cdots m_{k,H-1} = S_k P_I \tag{12}$$

An equilibrium point of the system, $n_e \triangleq (n_{ke}, \nu_{ke}, m_{kie} : 0 \leq k \leq J, 0 \leq i \leq H-1)$, is obtained as a solution to the set of simultaneous Eqs. (8)–(12). In order to find the equilibrium point n_e, we must first determine the value of $P_I(n_e)$. From Eq. (3), we can see that the expectation $E[P_I(n)]$ with respect to n is given by $\Pr[P_I(n) = 1]$, which is the probability that the channel is idle at the beginning of a slot. In the EPA, the expectation of a random variable with respect to n is approximated by the value of the random variable at the equilibrium point n_e. Therefore, we approximate $P_I(n_e)$ by $\Pr[P_I(n) = 1]$. The channel is idle at the beginning of a slot if and only if no user is in any S_{ki} mode (for $0 \leq k \leq J$ and $0 \leq i \leq H-1$). Then, we approximate the probability that a user is in S_{ki} mode by m_{kie}, the value of m_{ki} at n_e. Therefore, the probability that a user is in any S_{ki} mode is given by

$$P_{\mathrm{ST}} = \sum_{i=0}^{H-1} \sum_{k=0}^{J} m_{kie} \tag{13}$$

As already stated, $\sum_{i=0}^{H-1} \sum_{k=0}^{J} m_{ki} = 0$ or 1 and since $P_I(n_e) = 1 - P_{\mathrm{ST}}$, then

$$P_I(n_e) = 1 - H \sum_{k=0}^{J} S_k P_I \tag{14}$$

From Eqs. (8)–(12) and (14), we get the following equations:

$$M\sigma - \sigma \sum_{k=0}^{J} n_k = (\sigma G / P_I) + \sigma HGBP_I + (1-\sigma) P_I GB + (1-\sigma) n_J p_J (1 - BP_I) \tag{15}$$

$$n_0 = \frac{P_I GB}{[1 - (1 - BP_I)^{J+1}] p_0} \tag{16}$$

$$n_k = \frac{n_0 p_0 (1 - BP_I)^k}{p_k}, \quad 1 \leq k \leq J \tag{17}$$

where

$$B = \mathrm{e}^{-G} P_I + \mathrm{e}^{-(H+1)G}(1 - P_I) \tag{18}$$

and the probability P_I is the solution of the second-order polynomial equation:

$$HG(e^{-G} - e^{-(H+1)G})P_I^2 + (HGe^{-(H+1)G} + 1)P_I - 1 = 0 \quad (19)$$

The solution of the simultaneous Eqs. (15)–(19) determines the equilibrium point of the system. Setting $Y \triangleq e^{-G}$, we can solve the equations numerically. Since $0 < Y \leq 1$, we search the interval (0,1] for solutions. An algorithm that can be used to obtain the solutions is as follows [5]:

Step 1: $i = 0$; $Y^{(0)} = 1$ (for $G = 0$),
Step 2: calculate P_I from Eq. (19).
Step 3: calculate B from Eq. (18).
Step 4: calculate n_0 through n_J from Eqs. (16) and (17).
Step 5: calculate both sides of Eq. (15). If the magnitude of the difference between the left-hand and right-hand sides is smaller than some tolerance factor, then we assume that an equilibrium point is obtained.
Step 6: $i = i + 1$; $Y^{(i)} = Y^{(i-1)} - \varepsilon$, where $0 < \varepsilon \ll 1$
Step 7: if $Y^{(i)} \leq 0$, then stop.
Step 8: go to step 2.

We can have more than one solution by using the above algorithm. If there is only a single solution, the system is said to be stable; otherwise, it is unstable. In the unstable case, we can select the equilibrium point with the smallest value although this does not necessarily give a good estimate of the steady-state performance. Even in this case, however, we know the worst-case throughput–delay performance for some finite time period. Thus, the equilibrium point gives us useful information about the system stability and performance.

4. Performance analysis

Throughput $S(n)$ in state n is defined as the number of successful packet transmissions per H slots (i.e. the transmission time of a packet). Then, from Fig. 1, we have

$$S(n) = \sum_{i=0}^{H-1} \sum_{k=0}^{J} m_{ki} \quad (20)$$

From Eqs. (12) and (20), the throughput at the equilibrium point is expressed as

$$S(n_e) = H \sum_{k=0}^{J} S_k(n_e) P_I(n_e) \quad (21)$$

Substituting Eq. (7) into Eq. (21) and by using Eq. (6), we get

$$S(n_e) = HP_I(n_e)G(n_e)(e^{-G(n_e)}P_I(n_e) + e^{-(H+1)G(n_e)}(1 - P_I(n_e))). \quad (22)$$

We next evaluate the average message delay D, which is defined as the average time (in units of H slots) from the moment a packet is generated until the successful transmission of the packet is completed. In order to evaluate D, we first calculate the average number of total packets in the system, I, given by

$$I = \sum_{k=0}^{J} z_k \quad (23)$$

where z_k is the average number of users that have tried k times to transmit their packets. Since, each user has no more than a packet in his buffer, the number of packets in the system is the same with the number of users in TR_k, W_k, and S_{ki} modes (*for* $0 \leq k \leq J$ and $1 \leq i \leq H - 1$). Since the expectation of a random variable is approximated by the value of the random variable at an equilibrium point [5], then

$$z_k = n_k + \nu_k + (H - 1)m_k. \quad (24)$$

Substituting Eqs. (9) and (12) into Eq. (24) and using Eq. (7), we get

$$z_k = n_k + (n_k p_k / P_I) + (H - 1)n_k p_k B P_I \quad (25)$$

and finally,

$$I = G[1/P_I + (H - 1)BP_I] + \sum_{k=0}^{J} n_k \quad (26)$$

By using the Little's result, we can express the average packet delay (in units of H slots) as

$$D = \frac{I}{\bar{S}} \quad (27)$$

where $\bar{S}$ is the expectation of the throughput with respect to n. Substituting Eqs. (26) and (22) into Eq. (27), we get the expression for the average packet delay

$$D = \frac{G[1/P_I + (H - 1)BP_I] + \sum_{k=0}^{J} n_k}{HP_I GB} \quad (28)$$

Next, we determine the conditional probability, $P_{suc}(n)$, that a successful transmission occurs in a slot, given that the channel in state n is idle as

$$P_{suc}(n) = \sum_{k=0}^{J} S_k \quad (29)$$

Substituting Eq. (7) into Eq. (29) and using Eq. (18), we get that

$$P_{suc}(n) = GB \quad (30)$$

Let $P_{notran}(n)$ be the conditional probability that no user starts transmission in a slot, given that the channel in state n

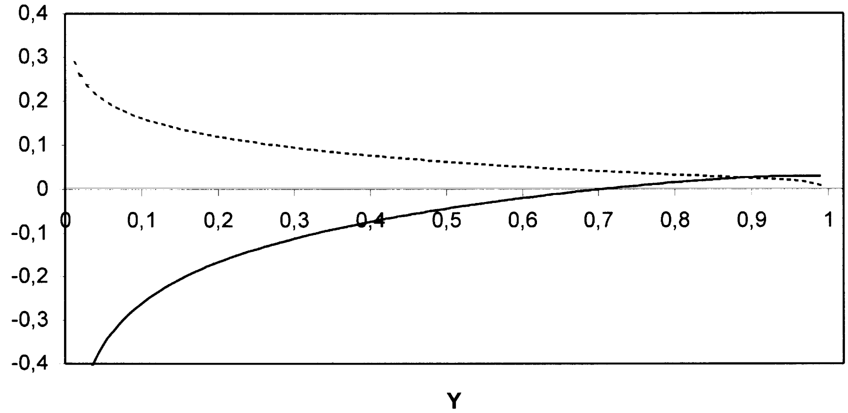

Fig. 2. Equilibrium point determination by solving the system simultaneous equations.

is idle, then

$$P_{\mathrm{notran}}(n) = \left[\prod_{i=0}^{J}(1-p_i)^{n_i}\right]PI + \left[\left(\prod_{i=0}^{J}(1-p_i)^{n_i}\right)^{H+1}\right] \times (1-PI) = \mathrm{e}^{-G}PI + \mathrm{e}^{-(H+1)G}(1-PI) \tag{31}$$

and using Eq. (18),

$$P_{\mathrm{notran}}(n) = B. \tag{32}$$

Then, the conditional probability, $P_{\mathrm{col}}(n)$ that a collision occurs in a slot, given that the channel in state n is idle, is

$$P_{\mathrm{col}}(n) = 1 - P_{\mathrm{notran}} - P_{\mathrm{suc}} = 1 - (G+1)B. \tag{33}$$

5. Numerical results

In this section, we use the above-derived equations in order to examine the system's performance in terms of throughput and packet delay. We also explore the effect of various parameters to the system performance by analyzing the system for several values of the number of users M and the packet length H. Furthermore; the stability behavior of the system is obtained from the number of the system equilibrium points. Fig. 2 shows the numerical results of applying the algorithm described in Section 3 to the two parts of Eq. (15) for all values of Y in the interval (0,1]. The system of equations results to a unique solution, thus yielding only one equilibrium point. Therefore, the system is considered stable.

Although the above analysis is generally applicable to any network data rate, in the rest of this section, we present some results by considering a 10 Mbps Ethernet with slot duration of 512 bits. The offered load to the system is $M\sigma H$ (packets per H slots). Figs. 3 and 4 display the throughput and the delay curves for various values of packet length and various numbers of users. The variation of throughput as a function of load is shown in Fig. 3. As long as the offered load remains low, for any number of users, the throughput equals the offered load since the inter-packet arrival time is sufficiently large which leads to infrequent collisions and minimal delay as shown in Fig. 4. As the offered load

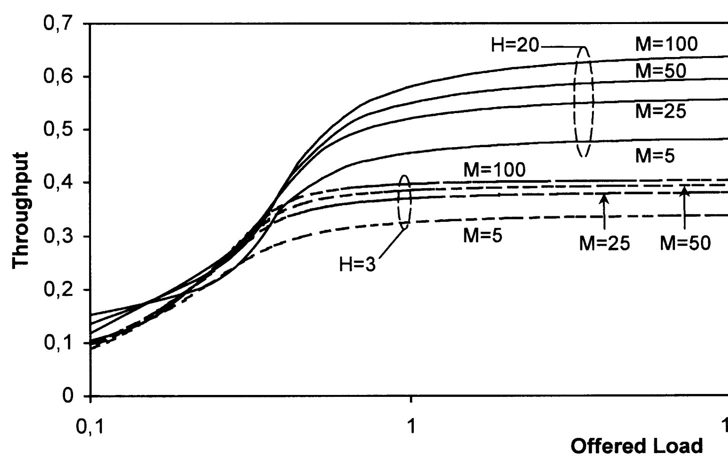

Fig. 3. Throughput versus offered load.

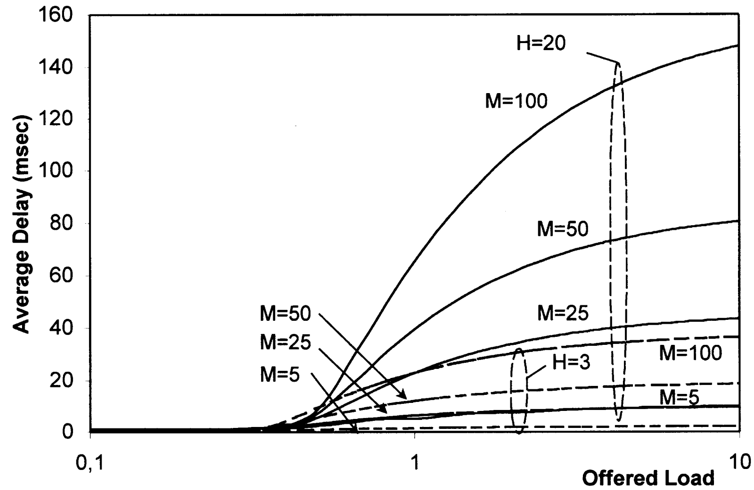

Fig. 4. Average packet delay versus offered load.

increases more stations have packets to transmit, causing an increase in the rate at which collisions occur. According to the truncated binary exponential backoff algorithm, some stations suffer multiple collisions and spend more time in the backlogged state, while other stations transmit successfully without collisions. This effectively reduces the number of stations contending for the channel, thus throughput continues to increase to a maximum value. For higher values of load, throughput drops to a stable value due to the fact that the mean time the stations are contending for the channel, or are backlogged, is much greater than the time the stations are awaiting for a new packet. Thus, increasing the load has a little effect to the throughput but increases the delay, as shown in Fig. 4.

The delay–throughput performance is shown in Fig. 5. For a fixed value of packet size H, the channel exhibits a maximum achievable throughput, which depends on the packet's size. Note that the corresponding delay–throughput performance for a given packet size is independent of the number of users. Each dashed line represents the case where the offered load and the number of users remains constant while the packet length changes. As the packet length increases while the offered load remains constant, less packet transmissions have to be performed, thus less competing periods occur.

In Fig. 6, the conditional probabilities $P_{\rm suc}(n)$, $P_{\rm notran}(n)$ and $P_{\rm col}(n)$ are shown as a function of M, the number of users. The load on the network is held constant at $M\sigma H = 1$ and the packet length is $H = 3$. Note that for a small number of users the probability of a successful transmission in a slot increases as the number of users is increased. For a large population network, the probability of a successful transmission increases slightly with the number of users. As the number of users increases, the probability that at least a user will have data to transmit also increases, thus the probability that no user starts transmission in a slot decreases and the collision probability increases. Note that the above probabilities are slightly affected by the number of users. These results explain the increase in the maximum value of throughput, when the number of users becomes larger, as was also observed in Fig. 3. Although the load

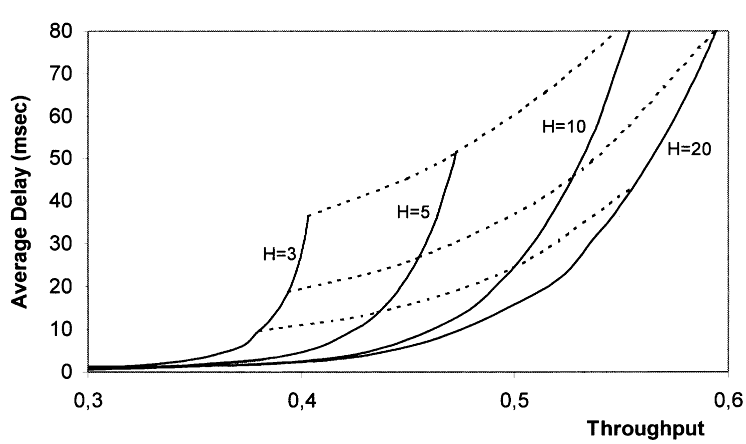

Fig. 5. Throughput versus delay for various packet lengths.

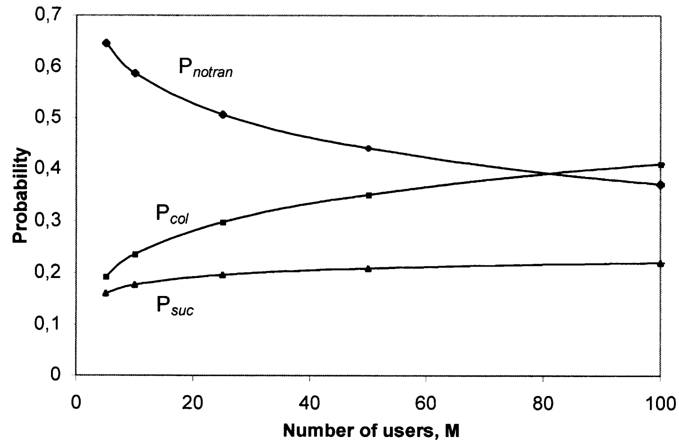

Fig. 6. P_{suc}, P_{notran} and P_{col} as a function of the number of users.

Table 1
Maximum throughput comparison

α	EPA analysis ($M = 400$)	Tobagi and Hunt ($M = \infty$)	Lam ($M = \infty$)
0.313 ($H = 3$)	0.40	0.429	0.332
0.05 ($H = 20$)	0.691	0.784	0.757

in the network is the same and the probability of a collision is higher, the idle slots that are more likely to happen in a small population network reduce the maximum throughput value.

The accuracy of the analytic results obtained from by our model is examined by comparing them with other 1-persistent CSMA/CD analytical models. We consider two analytical models of CSMA/CD that are commonly used for estimating Ethernet performance [2,3]. Lam analysed an infinite population model under the assumption that the system is stabilized using a suitable adaptive algorithm [2]. The arrival process is Poisson and the channel is considered slotted, with slot duration equal to two times the end-to-end propagation delay. Tobagi and Hunt presented a model assuming an infinite number of hosts generating packets with a Poisson distribution [3]. The channel is also considered slotted, where the slot size is the end-to-end propagation delay. Retransmission delays are assumed to be arbitrarily large. From the above models, the value of asymptotic throughput is recomputed and we compare the results for two different values of parameter α. In order to compare the analytic results from the infinite population models, we use a large value for M ($M = 400$) to our finite population model. Table 1 shows the comparison results for these models in terms of maximum throughput of a CSMA/CD network. We note that the results of our model are similar to the results of previously presented works.

Table 2
Throughput and average delay for $H = 3$ and $M = 400$

Offered load	Throughput		Average delay (ms)	
	Analysis	Simulation	Analysis	Simulation
0.04	0.042	0.042	0.26	0.21
0.10	0.10	0.29	0.10	0.25
0.22	0.22	0.22	0.42	0.40
0.34	0.33	0.36	2.28	1.37
0.40	0.37	0.42	9.10	4.90
0.52	0.40	0.48	31.92	29.81
0.70	0.40	0.48	57.82	62.50
0.82	0.40	0.48	64.26	70.76
1.00	0.40	0.48	83.51	83.15

We also studied the validity of the results obtained from our analytic model by comparing them with simulation results. We executed various simulations using the COMNET modeling tool by introducing the assumptions of our analytic model and the comparison results are presented in Table 2. It can be concluded that the above model can be used as an approximate prediction of the Ethernet performance for a finite number of users, taking also into account the influence of the binary exponential backoff algorithm.

6. Conclusions

In this paper, we presented an analysis of the binary exponential backoff algorithm in a slotted 1-persistent CSMA/CD system for the single buffer case. We developed a Markovian model of the system and we analyzed the model using the approximate technique of the EPA. The presented numerical results indicate that at least under some conditions the system is stable and that the analytic results are accurate compared to the simulation results and the analytic results of previously published works.

Appendix A

Approximation of Eq. (4) is valid for small values of p_i and is usually used to simplify the analysis. In our model and for very small values of i, this approximation cannot be considered accurate, but as it is shown in the rest of this appendix, it does not introduce significant error in the final system results.

The conditional probability $S_k(n)$ (Eqs. (1) and (2)) without the approximation (4) is expressed by the following equations:

$$S_{01}(n) = \left[\sigma\left(M - \sum_{i=1}^{J} n_i\right)(1-\sigma)^{M - \sum_{i=1}^{J} n_i - 1} \prod_{i=1}^{J} (1-p_i)^{n_i} \right] \times P_I, \quad (k=0) \tag{A1}$$

$$S_{k1}(n) = \left[n_k p_k (1-p_k)^{n_k - 1} \left(1 - \frac{S_{01}}{P_I}\right) \prod_{i=1, i\neq k}^{J} (1-p_i)^{n_i} \right] \times P_I, \quad (1 \le k \le J) \tag{A2}$$

and

$$S_{02}(n) = \left[\sigma\left(M - \sum_{i=1}^{J} n_i\right)(1-\sigma)^{M - \sum_{i=1}^{J} n_i - 1} \prod_{i=1}^{J} (1-p_i)^{n_i} \times \left(\prod_{i=1}^{J} (1-p_i)^{n_i} \right)^{H} \right](1-P_I), \quad (k=0) \tag{A3}$$

$$S_{k2}(n) = \left[n_k p_k (1-p_k)^{n_k - 1} \left(1 - \frac{S_{02}}{1-P_I}\right) \prod_{i=1}^{J} (1-p_i)^{n_i} \times \left(\prod_{i=1}^{J} (1-p_i)^{n_i} \right)^{H} \right](1-P_I), \quad (1 \le k \le J) \tag{A4}$$

Approximation (4) mainly affects the factor S_kP_I. In Table 3, we compare the results from Eqs. (A1)–(A4) to the results of the simplified equation for $H = 4$, $M = 100$ and load = 1. We note that this assumption has not significant effect on the system results. The system model can also be solved using Eqs. (A1)–(A4) and numerical methods and in this case, the results are similar to the results presented in this paper. Assumption (4) helps the derivation of an analytical expression for the system throughput and delay with a small loss to the accuracy of the results.

Table 3
Comparative Results

k	S_kP_I (simplified expression)	S_kP_I
0	0.023	0.084
1	0.018	0.032
2	0.014	0.017
3	0.011	0.011
4	0.90×10^{-2}	0.82×10^{-2}
5	0.71×10^{-2}	0.63×10^{-2}
6	0.56×10^{-2}	0.49×10^{-2}
7	0.44×10^{-2}	0.38×10^{-2}
8	0.35×10^{-2}	0.30×10^{-2}
9	0.27×10^{-2}	0.2×10^{-2}
10	0.21×10^{-2}	0.18×10^{-2}
11	0.17×10^{-2}	0.14×10^{-2}
12	0.13×10^{-2}	0.11×10^{-2}
13	0.10×10^{-2}	0.92×10^{-3}
14	0.84×10^{-3}	0.72×10^{-3}

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