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Computationally efficient method for tone detection in noisy channels

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A simple, efficient and reliable method is proposed for tone detection in noisy channels. The method is based on the fact that the information contained in a generated sequence of samples mainly depends on the relation between the sampling rate and the analogue signal bandwidth. This relation can also be used to implement filtering functions in frequency detection systems. The method is explicitly analysed and detailed mathematical formulae are derived for its basic parameters. The method has been implemented in vehicle mounted pagers and its implementation is described in detail. The main advantages of the proposed method are its simplicity, its requirements for low processing power due to its computational efficiency and its implementability using conventional microcontrollers.

1. Introduction

The problem of detecting a known sinusoidal signal with random phase and corrupted with additive gaussian noise is of major importance in many application areas such as in radar/sonar signal processing, in digital communications for demodulation of received signals corrupted with noise, etc. (Proakis 1989, Walker 1990, Root 1987). Owing to this wide applicability of frequency detection, a variety of methods that are optimum in some sense have been developed so far. These methods are generally applicable and can be successfully used in an M-ary signalling system where the receiver detects which one of the M possible frequencies has been received. The use of a bank of matched filters (Van Trees 1968), the detection by autoregressive spectral analysis (Kay 1982), and the frequency detection by zero-crossings (Kedem 1986) are some of the currently used methods in frequency detection systems. However, it is well known that in cases where either the input SNR is very low or the signals to be discriminated are closely spaced in the frequency band, all of the above methods require extra computations in order to perform satisfactorily. Thus, for instance, in autoregressive analysis a large number of data are required to compensate for the low SNR which causes an increase in the probability of error (Kay 1982). In frequency detection by zero-crossings, a satisfactory performance in very noisy conditions can be ensured only if zero crossing counts are used together with linear filtering (Kedem 1986). As for the bank of matched filters method, which maximizes the output SNR, it should be pointed out that in addition to its considerable computational burden, it also exhibits an unwanted inflexibility to variation in the noisy environment (Stein and Jones 1986).

Although the proposed method is a suboptimal one, it turns out to be quite successful in detecting a sinusoidal signal in a noisy environment. Moreover, its simplicity and its very low computational complexity make this method attractive for implementation using conventional technology.

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The method developed is appropriate for those applications, of which there are many, in which only one particular tone is to be detected. However, it could be used in a general M-ary signalling system as well; in which case the detector consists of a bank of subsystems like the proposed one, each tuned to detect one particular frequency.

In § 2, the problem formulation is described and its constraints are highlighted. The proposed method is analysed in § 3 where analytic expressions for the decision and the validation variables are developed and a sequential variant of the method is given. The implementation of the method is given in § 4; § 5 summarizes the basic characteristics and the results of the proposed method.

2. Problem formulation and method description

Various sequential tone signalling systems have been developed so far and they have been extensively used in many applications, ranging from simple remote control up to vehicle mounted pagers. The most well-known coding format is the so called 'hexadecimal sequential code' (HSC) which permits address codes and instruction codes to be exchanged between all units of a communication network under base, mobile and personal communication schemes. The main advantage of the HSC is that it ensures signalling integrity under poor and variable communication conditions. The HSC format employs a set of tone frequencies and the codes are transmitted as a sequence of single tones which represent a series of alphanumeric numbers. There are four international standards (CCIR, EEA, ZVEI and EIA for mapping the hexadecimal characters to the frequency domain and they also have different frequency tolerance and tone duration. The HSC format is also used for '5-tone' selective calling which is mainly used to implement the signalling of voice channels in radio systems. For each of the above mentioned standards, a separate IC, hardwired for a standard format, is used to implement the tone detection function. Thus, more than one IC must be used to support various standards (this increases the hardware complexity and cost) and the system central processor must be involved in order to translate the semantics of the detected tone received from the specificic IC, to semantics comprehensible by the application. Starting from these requirements, a simple method for '5-tone' selective calling has been developed that can be implemented using a conventional microcontroller. It can easily handle the various tone sets and the switching function from one format to another can be performed 'on-the-fly'. In the rest of this section, the method used to detect a single tone is described, while the total code detection is given in the implementation description. So the problem which must be initially solved is how the existence of a specific frequency (tone), with short tolerance limits and under noisy conditions, is detected, using an algorithm which is simple enough to be implemented in low cost hardware.

The method used to detect the tone is based on the following technique. It is well known that the averaging of periodic signals can be used to eliminate the additive gaussian noise from the signal. In frequency detection systems, all frequencies except the desired one are considered to be noise for the system and must be removed. This can be achieved by adjusting the sampling rate to be a multiple of the expected frequency and then averaging the received samples using the samples of the same phase. After the end of this procedure, there are only a small number of data to be processed, while these data contain all the information needed to detect the expected frequency. The block diagram of the tone decision procedure is shown in Fig. 1. Once the data have been processed, autocorrelation and cross-correlation-like techniques are used for decision and validation purposes, respectively, as will be explained in detail in the next section. Figure 2 shows an example of the implementation of the proposed method to a signal which contains the expected frequency corrupted with noise. After the sampling and the averaging processes, the samples of a period of the desired frequency are regenerated, while the additive noise has been reduced.

As has been mentioned in the previous section, the problem treated here is a constrained variant of that encountered in a general M-ary signalling system. In the latter, the receiver has to decide which one of the M possible sinusoidal signals has been sent. The phase of the received signal is random and the additive noise is considered to be of zero mean, white gaussian noise (Proakis 1989, Stein and Jone 1986). The aim in the following analysis is to develop a method capable of deciding whether a particular predetermined frequency, among M possible ones, is present in the received signal or not. That is, the receiver is tuned to detect the desired frequency while rejecting the others. As before, the phase is random and the signal is corrupted by additive zero mean, white gaussian noise. The received signal is sampled and the resulting sequence of samples is given by

$$s(nT_s) = \cos(2\pi f_m T_s n + \varphi) + v(nT_s) \tag{1}$$

where f_m is the frequency of the received signal, T_s is the sampling period, $v(nT_s)$ is a zero mean, white gaussian noise with variance σ_v^2 and φ is the initial phase which is assumed to be a uniformly distributed random variable, i.e.

$$\Pr(\varphi) = \frac{1}{2\pi} \qquad 0 \le \varphi \le 2\pi \tag{2}$$

For the sake of simplicity and without loss of generality, the amplitude of the sinusoidal signal is assumed to be equal to one with the noise power scaled accordingly. Let f_d be the frequency of the desired tone. The analysis of the procedure by which it is decided whether $\cos(2\pi f_d t)$ is present or not is described in the next section.

3. Analysis of the method

The constraint that a particular tone has to be detected will be exploited in the sampling process. The sampling rate f_s, is chosen to be equal to a multiple integer of

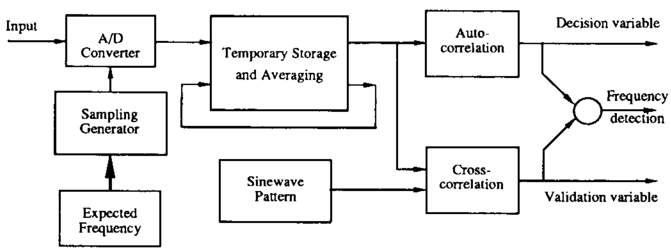

Figure 1. Block diagram of the tone detection procedure.

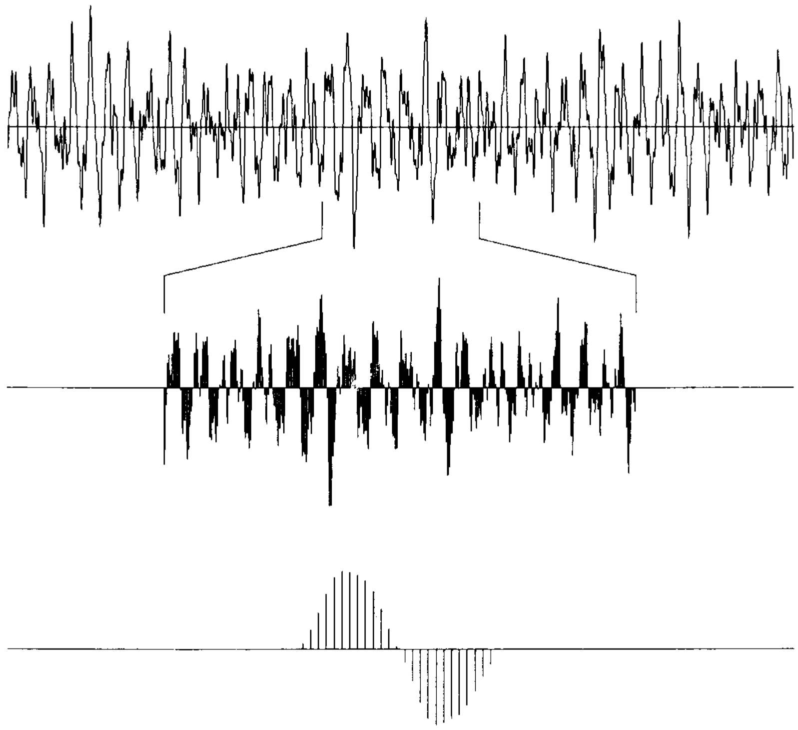

Figure 2. Example of the proposed method: (*a*) the received signal; (*b*) the sampling process; (*c*) the averaging results.

the desired frequency f_d, i.e. $f_s = Lf_d$. That is, the first L samples cover a time interval equal to the period of the desired tone. These first L samples will be considered as the representative, because each is taken to be the initial sample of a set of samples with distance LT_s between each other. Let f_m be the frequency of the received sinusoidal signal; then taking into account the above discussion, the sampled sequence can be expressed as:

$$s(k, n) = \cos(2\pi f_m L T_s n + \varphi_k) + v(k, n) \quad k = 0, 1, \ldots, L-1, \quad n = 0, 1, \ldots \tag{3}$$

where $v(k,n)$ is the additive white noise, with probability density function $N(0, \sigma_v^2)$ and φ_k is given by

$$\varphi_k = 2\pi \frac{T_s}{T_m} k + \theta \quad k = 0, 1, \ldots, L-1 \tag{4}$$

with θ being the initial phase of the input signal, uniformly distributed between 0 and 2π. In the next step a type of averaging on the received sequence of samples given by (3) is performed, according to the following method. Each of the representative samples $s(k, 1)$, $k = 0, 1, \ldots, L-1$, is summed with the $N-1$ subsequent samples of the respective set, i.e. with the samples $s(k, L)$, $s(k, 2L)$, $\ldots$, $s(k, (N-1)L)$, and the resulting sum is divided by N. Thus, the L averaged samples are given by:

$$\bar{s}(k)=\bar{x}(k)+\bar{v}(k) \quad k=0, 1, \ldots, L-1 \tag{5}$$

where

$$\bar{x}(k)=\frac{1}{N}\sum_{n=0}^{N-1} \cos\left(2\pi\frac{f_m}{f_s} LT_s n+2\pi\frac{f_m}{f_s} k+\theta\right) \tag{6}$$

and

$$\bar{v}(k)=\frac{1}{N}\sum_{n=0}^{N-1} v(k, n) \tag{7}$$

The variable $\bar{v}(k)$ is the sum of N statistically independent gaussian random variables; it is therefore also a gaussian random variable with zero mean and variance equal to σ_v^2/N.

Define P_s as the power of the averaged samples, i.e.

$$P_s=\frac{1}{L}\sum_{k=0}^{L-1} |\bar{s}(k)|^2 \tag{8}$$

From (5) and (8), we have

$$P_s=P_{xx}+P_{xv}+P_{vv} \tag{9}$$

where

$$P_{xx}=\frac{1}{L}\sum_{k=0}^{L-1} |\bar{x}(k)|^2 \tag{10}$$

$$P_{xv}=\frac{2}{L}\sum_{k=0}^{L-1} \bar{x}(k)\bar{v}(k) \tag{11}$$

$$P_{vv}=\frac{1}{L}\sum_{k=0}^{L-1} |\bar{v}(k)|^2 \tag{12}$$

Note that the value of P_s will be used as the system decision variable. As will be shown in the following analysis, P_s attains its maximum value for $f_m=f_d$, while for frequencies outside the neighbourhood of f_d its magnitude is below some predetermined threshold. In the following, an analytical formula for P_s will be derived which reveals its role as a desision variable.

From the definition of $\bar{x}(k)$ in (6) and after some mathematical manipulations, it is obtained that

$$\bar{x}(k)=\frac{1}{N}\left[\frac{\sin\left(\pi LN\frac{f_m}{f_s}\right)}{\sin\left(\pi L\frac{f_m}{f_s}\right)}\right]\cos\left(\pi L(N-1)\frac{f_m}{f_s}+2\pi\frac{f_m}{f_s} k+\theta\right) \tag{13}$$

Substituting (13) in (10), after some calculations, we have that

$$P_{xx}=\frac{1}{2N^2}\left[\frac{\sin\left(\pi LN\frac{f_m}{f_s}\right)}{\sin\left(\pi L\frac{f_m}{f_s}\right)}\right]^2\left[\frac{1}{L}\left[\frac{\sin\left(2\pi L\frac{f_m}{f_s}\right)}{\sin\left(2\pi\frac{f_m}{f_s}\right)}\right]\cos\left(2\pi(LN-)\frac{f_m}{f_s}+2\theta\right)+1\right] \quad (14)$$

In the noiseless case, P_s is equal to P_{xx}; it is thus interesting to see the form of P_{xx}. Figure 3 shows the value of P_{xx} versus frequency for a typical example in which $f_d=1275$ Hz, $L=8$, $N=32$ and $\theta=0$. Setting the threshold at 0·4, it can be seen that when $P_{xx}>0{\cdot}4$ any frequency in the interval $f_d\pm0{\cdot}5\%$ is identified as f_d, while any frequency out of this interval gives $P_{xx}<0{\cdot}4$, and the decision is that the desired frequency has not been received. Increasing L and N, the acceptance interval of f_d becomes smaller, for a given threshold, and therefore the performance of the method is improved.

In the noisy case, the form of the decision variable is affected by the additive noise, as can be seen from the presence of the last two terms in relation (9). In order to see the influence of noise in the proposed procedure, analytical expressions for P_{xv} and P_{vv} must first be derived.

From the definition of P_{xv} in (11) and using (13), we have

$$P_{xv}=\frac{2}{LN}\left[\frac{\sin\left(\pi LN\frac{f_m}{f_s}\right)}{\sin\left(\pi L\frac{f_m}{f_s}\right)}\right]\sum_{k=0}^{L-1}\cos\left(\pi L(N-1)\frac{f_m}{f_s}+2\pi\frac{f_x}{f_s}k+\theta\right)\bar{v}(k) \quad (15)$$

The summation term in (15) is defined as

$$\bar{v}_x=\sum_{k=0}^{N-1}\cos\left(\pi L(N-1)\frac{f_m}{f_s}+2\pi\frac{f_m}{f_s}k+0\right)\bar{v}(k) \quad (16)$$

Since $\bar{v}(k)$, $k=0, 1, \ldots, L-1$, for a given frequency f_m are independently identically distributed (i.i.d.) gaussian random variables with p.d.f. given by $N(0, \sigma_v^2/N)$, it is readily deduced that $\bar{v}_x$ is also gaussian with zero mean and variance equal to

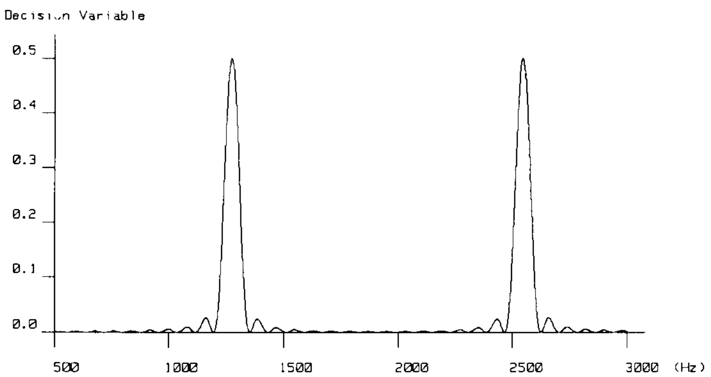

Figure 3. Decision variable versus frequency (expected frequency: 1275 Hz).

$$\sigma_{\bar{v}_x}^2 = \frac{\sigma_v^2}{N} \sum_{k=0}^{L-1} \cos^2\left(\pi L(N-1)\frac{f_m}{f_s} + 2\pi \frac{f_m}{f_s} k + \theta\right) \tag{17}$$

which after some lengthy manipulations can be further simplified to

$$\sigma_{\bar{v}_x}^2 = \frac{\sigma_v^2 L}{2N} + \frac{2\sigma_v^2}{N} \left[\frac{\sin\left(2\pi L \frac{f_m}{f_s}\right)}{\sin\left(2\pi \frac{f_m}{f_s}\right)}\right] \cos\left(2\pi\,(LN-1)\frac{f_m}{f_s} + 2\theta\right) \tag{18}$$

Having computed the mean and variance of $\bar{v}_x$, the respective moments of P_{xv} can be derived using (15). It can easily be seen that, for a given frequency f_m, the variable P_{xv} is also gausian with zero mean and variance given by

$$\sigma_{xv}^2 = \sigma_v^2 \frac{2}{LN^3} \left[\frac{\sin\left(\pi LN \frac{f_m}{f_s}\right)}{\sin\left(\pi L \frac{f_m}{f_s}\right)}\right]^2 \left[1 + \frac{4}{L}\left[\frac{\sin\left(2\pi L \frac{f_m}{f_s}\right)}{\sin\left(2\pi \frac{f_m}{f_s}\right)}\right] \cos\left(2\pi\,(LN-1)\frac{f_m}{f_s}\right) + 2\theta\right] \tag{19}$$

It remains to investigate the statistical properties of the variable P_{vv}. Note that this term is due thoroughly to the presence of noise in the input signal. Recall from the definition of $\bar{v}(k)$ in (7) that these random variables, for a given frequency f_m, and for $k=0, 1, \ldots, L-1$ are i.i.d. gaussian random variables with zero mean and variance σ_v^2/N. Taking into account this result and using (12), it can easily be proved that P_{vv} is a random variable with a probability density function that is a chi-square with L degrees of freedom (Proakis 1989, Van Trees 1968). Its p.d.f. is given by

$$p(P_{vv}) = \frac{1}{\sigma^L 2^{L/2} \Gamma(L/2)} P_{vv}^{(L/2)-1} \exp\left(-P_{vv}/2\sigma^2\right) \quad P_{vv} > 0 \tag{20}$$

where $\Gamma(L/2)$ is the gamma function. It is straightforward to derive the following relation for the mean and variance of P_{vv}:

$$\mu_{vv} = \frac{1}{N} \sigma_v^2 \tag{21}$$

$$\sigma_{vv}^2 = \frac{2}{LN^2} \sigma_v^4 \tag{22}$$

From the above derivations for the random terms P_{xv} and P_{vv}, we are now able to proceed with a more detailed analysis of the detection procedure. From the presence of the term LN^3 in the denominator of (19) it can be easily verified that, for typical values of L, N and even for low SNR (up to -5 dB), the variance σ_{xv}^2 takes very small values, less than 10^{-5}. Combining this result with the fact that P_{xv} is of zero mean, it can be deduced that the influence of this random term on the value of the decision variable P_s is almost negligible. For that reason it will not be taken into account in our further discussion.

From the analysis so far, it is evident that the rariable P_s can be used as the decision variable in the detection procedure. Since the influence of P_{xv} is negligible, it is deduced that the performance of P_s depends mainly on P_{xx}, given by (14), as well as on the random variable P_{vv} whose influence can be analytically computed using (21) and (22).

In order to set the threshold, the minimum tone decode bandwidth (MTDBW) must first be specified. Any tone within the MTDBW is considered to be valid (i.e. the desired one) while any other tone outside the MTDBW is rejected as invalid. Certainly, the MTDBW must be less than the minimum distance between any two of the M possible frequencies. On the other hand, it must be greater than the frequency tolerance that inevitably appears in tone transmission. Once the MTDBW has been specified, the threshold can be determined so that the decision can be made with minimum probability of error. The value of the decision variable versus the frequency variation from the expected one for various values of the noise level and the sampling duration is shown in Fig. 4. From Fig. 4(a), which shows the noiseless case, it is obvious that the accuracy of the method increases as long as the sampling duration increases, and for large values of N the method behaves like a very sharp

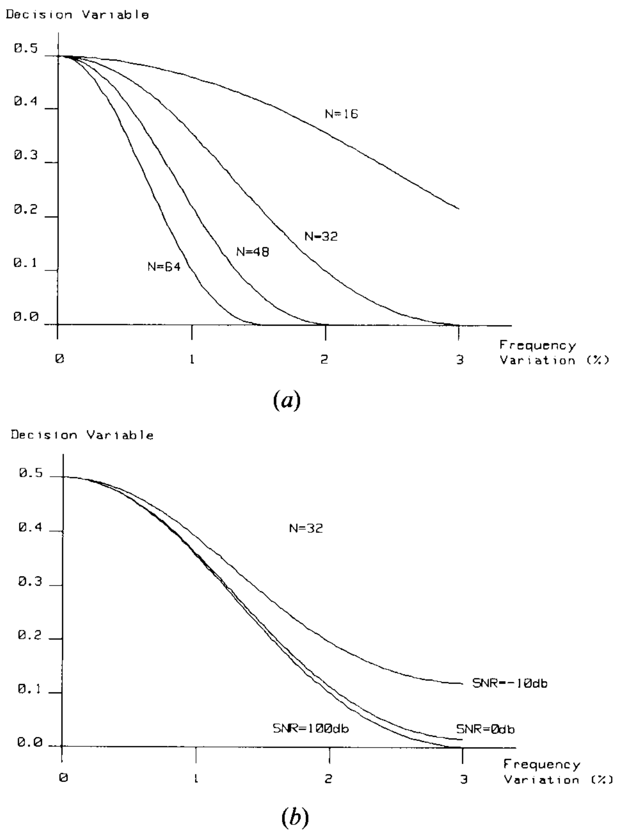

Figure 4. The value of the decision variable versus the frequency variation: (a) for various values of N; (b) for various SNR values.

narrow band filter. On the other hand, the method satisfies the requirements of international coding standards, even for small sampling duration, since its accuracy is greater than the standards acceptable accuracy. The influence of the noise level is shown in Fig. 4(*b*). Up to the value of 0 dB SNR for $N=32$, there is almost no variation of the value of the decision variable. Below this value, the accuracy of the method is seriously affected and its acceptability depends on the application requirements. For the previously mentioned HSC systems, the proposed method satisfies their requirements and can be used effectively.

Obviously, for a given threshold the probability of error depends on the parameters L and N as well as on the variance σ_v^2 of the additive noise. Specifically, as the parameters L and N increase the probability of error decreases. It should be noted that the only limitation with respect to L and N is the duration of the transmitted tone signal; i.e. the total sampling time NLT_d must be less than the tone duration.

Since the p.d.f. of P_{vv} has been found to be chi-squared with mean value and variance given by (21) and (22), it is deduced that the probability of an erroneous decision can be computed analytically. However a rough estimate can easily be derived by using Chebyshev inequality or other related tools from probability theory (Proakis 1989). Using these tools, the probability of error was found to be quite satisfactory. For instance, for typical values of L, N and SNR (e.g. $L=8$, $N=32$, SNR > 10 dB) the probability of error is of the order of 10^{-6}.

3.1. *Rejecting the harmonics*

It can easily be verified that the procedure described, as developed so far, cannot decide whether the detected frequency is the expected one, f_d, or some of its harmonics. Thus, in case there are some harmonically—or nearly harmonically—related frequencies among the M possible ones, a misleading decision is likely to be made. This is due to the imposed restriction that the sampling rate is chosen to be equal to a multiple integer of the desired frequency f_d.

To tackle the above problem, a second decision variable, called the validation variable is defined as

$$C_s=\frac{1}{L}\sum_{k=0}^{L-1}|\bar{s}(k)\cos(2\pi f_d kT_s+\varphi)|$$

$$=\frac{1}{L}\sum_{k=0}^{L-1}\left|\bar{s}(k)\cos\left(2\pi\frac{k}{L}+\varphi\right)\right| \tag{23}$$

i.e., the received signal $s(k,n)$ is averaged and the outcome $\bar{s}(k)$, given by (5)–(7), is cross-correlated with the desired sinusoidal signal. Recall that the amplitude of the involved sinusoid was assumed unity. Variable φ is the starting phase of the ideal tone signal and it is a random variable uniformly distributed between 0 and 2π. Obviously, the less the relative distance between the received frequency f_m and the desired frequency f_d the greater the value of C_s. In the case where $f_x=f_d$ and $\theta=\varphi$, the variable C_s attains its maximum value. It can be shown that in the noiseless case, variable C_s is given by the following expression:

$$C_s=\frac{1}{2LN}\left[\frac{\sin\left(\pi LN\frac{f_m}{f_s}\right)}{\sin\left(\pi\frac{f_m}{f_s}+\frac{\pi}{L}\right)}\right]\cos\left(\pi\,(LN-1)\frac{f_m}{f_s}-\frac{\pi}{L}+\theta+\varphi\right)$$

$$+\left[\frac{\sin\left(\pi LN\frac{f_m}{f_s}\right)}{\sin\left(\pi\frac{f_m}{f_s}-\frac{\pi}{L}\right)}\right]\cos\left(\pi(LN-1)\frac{f_m}{f_s}+\frac{\pi}{L}+\theta+\varphi\right) \qquad (24)$$

In Fig. 5 the plot of C_s versus frequency is given ($L=8$, $N=32$, $f_d=1275\,\mathrm{Hz}$, $\theta=\varphi=0°$). From this figure it can be seen that C_s exhibits a peak at $f=f_d$, while outside the neighbourhood of f_d its value tends to zero for any frequency, even for a harmonic one. Since C_s is used only for validation reasons, the general noisy case is not detailed here (as was necessary for the decision variable P_s). Besides, the presence of noise changes the form of C_s only slightly.

Note that there will be some bias in the positioning of the peak of C_s as well as in its maximum value, due to the fact that the received signal is not in phase with the ideal one. However, this problem can easily be overcome by roughly estimating the value of the initial phase of the received signal during the sampling process.

3.2. *A sequential variant of the method*

From (14) and (19)-(22), it is evident that the parameter N plays a critical role in the detection procedure. The greater the parameter N, the less the probability of an erroneous decision. In order to cope with the worst case of very low SNR, the parameter N must be chosen to be large enough to increase data collection time, which is unnecessary for medium and high SNRs. To overcome this problem, a simple sequential procedure is proposed which adapts the value of N (therefore also the computing and data collection times) to the time varying SNR.

Let us define the variable

$$\bar{y}(k,t)=\frac{1}{t}\sum_{n=0}^{t-1}s(k,n) \quad k=0,1,\ldots,L-1, \quad t=0,1,\ldots,N_{\max} \qquad (25)$$

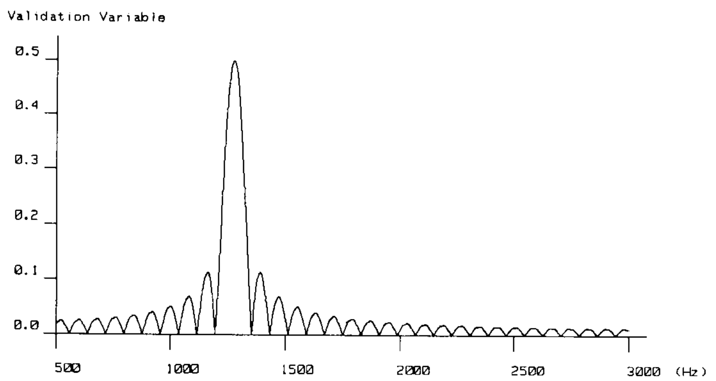

Figure 5. Validation variable versus frequency (expected frequency: 1275 Hz).

that is, each of the representative samples $s(k, 1)$, $k=0, 1, \ldots, L-1$, is now summed with $t-1$ subsequent samples of the respective set of samples instead of $N-1$ ones, as was the case before. The averaged samples are updated each time a new sample is received according to the following easily established relation:

$$\bar{y}(k, t)=\left(\frac{t-1}{t}\right)\bar{y}(k, t-1)+\frac{1}{t}s(k, t-1) \quad k=0, 1, \ldots, L-1; \quad t=0, 1, \ldots, N_{max} \quad (26)$$

For each t, the decision and the validation variables $P_s(t)$ and $C_s(t)$, respectively, are computed. The procedure terminates when a safe decision has been made. It is evident that this sequential variant of the method exhibits an increased computational burden because the decision parameter is computed for each new group of samples. However, this would be very useful in cases where the SNR is time varying.

4. Implementation of the method

As previously mentioned, the presented method constitutes the basic module of a '5-tone' selective calling system; so, before starting the description of the method's implementation, its use in the complete system design must be described.

Five-tone selective calling consists of five sequential tones of constant duration (t_0) and inter-tone (t_g) which depend on the coding format. An example of a 5-tone signal with details of the method's application is shown in Fig. 6. The calling command has a specific pattern for each pager, so this specific pattern must be sequentially detected. Each character of this specific pattern can be substituted by a 'group' character; so, for the duration of a tone at least two tone-detection processes must be executed. As will be shown later, the duration of the tone detection process is variable and depends on the expected frequency; this has to be taken into account in the determination of the implementation parameters. Furthermore, there is no way to determine the start of the first tone explicitly; so this ambiguity must also be considered. As is shown in Fig. 6, if t_{fx} is the duration of the tone detection process for frequency f_x, and t_{fa} is the respective duration for the 'group' frequency, then the implementation must satisfy the following inequality:

$$t_{fx}+t_{fa}+\max(t_{fx}, t_{fa}) \leq t_0 \quad (27)$$

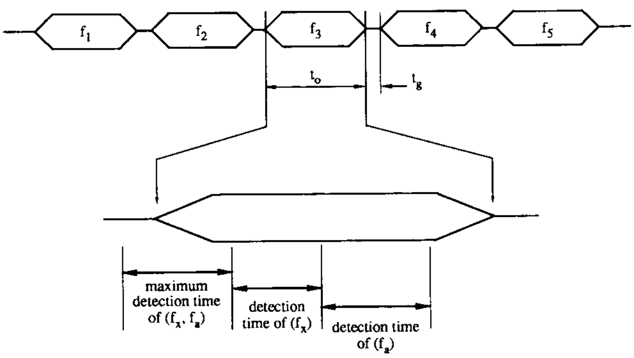

Figure 6. The '5-tone' selective signalling system.

Only under this condition can the system guarantee the reliability of the method. Figure 7 shows the flowchart of the 5-tone calling algorithm. The tones are detected sequentially and the 'command detected' signal is generated only if the pattern is detected in the proper time frame. It is obvious that the tone detection method for a specific character is executed repetitively using the 'character' and the 'group' frequencies under the supervision of an internal timer.

The discussion up to now results in the following functional requirements from the integrated circuit which will implement the method:

(*a*) A/D converter;
(*b*) high resolution timer systems;
(*c*) real-time interrupt circuit;
(*d*) internal RAM for data storage;
(*e*) internal EEPROM for code format tables; and
(*f*) serial interface for EEPROM update (e.g. for new codes).

In order to fulfil these requirements using a single chip, a microcontroller of the MC68HC11 family of Motorola Inc. was used. The microcontroller includes an 8-bit A/D converter with 16 μs conversion time for a single channel—a speed more than adequate for this application. In HSC systems, the frequency varies from 459 Hz (EIA) up to 2800 Hz (ZVEI). The block diagram of the method implementation is

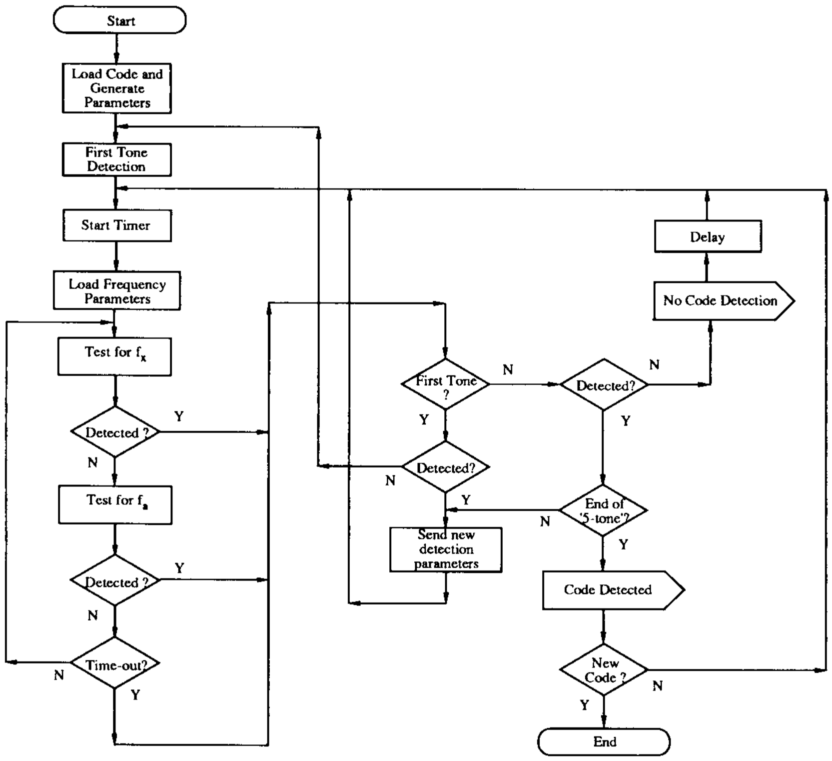

Figure 7. Flowchart of the '5-tone' selective signalling system algorithm.

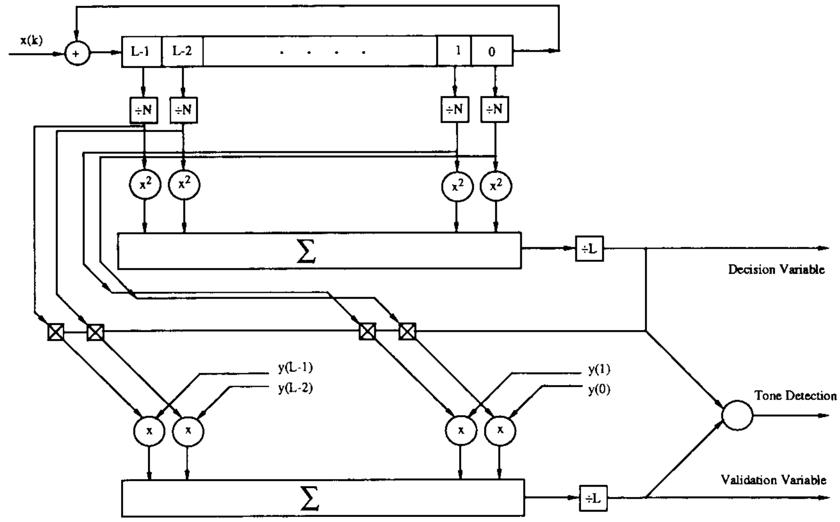

Figure 8. Block diagram of the algorithm implementation.

shown in Fig. 8. The method requires L samples per period for a total of N periods of the expected frequency. In the internal RAM of the microcontroller, a logical shift-register (1×16 bits) is used where the summation of the received samples is performed during the sampling process. When the data acquisition terminates, the averaging is performed by dividing each sum by the number of the sampled periods. In order to simplify the division function, which is the most time consuming instruction in the microcontroller, the value of N must be selected to be a power of two, so the division is performed using a small number of 'logical shift right' instructions. Having these L averaged values, which represent the values of a period, the autocorrelation process begins. The result of the autocorrelation process is compared with a predetermined threshold and the decision about the presence of the expected frequency or any of its harmonics is made. Having a positive result of that decision, the validation procedure begins. Initially the determination of the position of the 0° phase sample (or the nearest to it) is performed and then the cross-correlation process is executed. The value of the 'validation variable' determines the existence of the expected frequency.

The time required to execute the tone detection process is variable and depends on the number of samples per period, the number of sampled periods and the expected frequency. It is equal to

$$t_{fx} = t_{aqu} + t_{aut} + t_{crc} \tag{28}$$

where the data acquisition time, t_{aqu}, is

$$t_{aqu} = T_f N \tag{29}$$

the averaging and autocorrelation time, t_{aut}, is

$$t_{aut} = L \log_2 N \; t_{sh} + L t_{mul} + (L-1) t_{ad} + t_{div} \tag{30}$$

Char-acter	CCIR				EEA				ZVEI				EIA			
	N	L	t_{aqu} (ms)	t_{fx} (ms)	N	L	t_{aqu} (ms)	t_{fx} (ms)	N	L	t_{aqu} (ms)	t_{fx} (ms)	N	L	t_{aqu} (ms)	t_{fx} (ms)
0	32	24	16·15	17·88	16	24	8·07	9·63	32	16	13·33	14·45	4	64	6·66	10·18
1	32	20	28·46	29·91	8	44	7·12	9·98	16	44	15·09	17·95	4	64	5·39	8·91
2	32	40	26·73	29·58	8	40	6·68	9·28	16	40	13·79	16·39	4	50	4·54	7·29
3	32	38	25·09	27·75	8	38	6·27	8·55	16	38	12·59	15·06	8	48	7·82	10·70
4	32	34	23·56	25·94	8	64	5·89	9·73	16	34	11·43	13·64	8	40	6·87	9·27
5	32	30	22·13	24·23	8	64	5·53	9·37	16	30	10·46	12·41	8	36	6·13	8·29
6	32	30	20·77	22.87	16	24	10·39	11·95	16	28	9·58	11·40	8	32	5·53	7·45
7	32	28	19·51	21·47	16	28	9·75	11·57	16	48	8·74	11·86	16	14	10·08	10·99
8	32	26	18·32	20·14	16	26	9·16	10·85	32	24	16·00	17·68	16	26	9·26	10·95
9	32	24	17·20	18·88	16	24	8·60	10·16	32	20	14·54	15·94	16	24	8·56	10·12
A	32	16	13·33	14·50	8	44	7·58	10·22	32	16	11·43	12·55	16	20	7·43	8·73
E	16	48	18·33	21·45	8	44	9·16	11·80	8	48	10·81	13·69	—	—	—	—

Tone detection timing for the various coding formats.

while the phase determination and cross-correlation time, t_{crc}, which participates only if the autocorrelation gives a positive result, is

$$t_{crc} = Lt_{ph} + Lt_{mul} + (L-1)t_{ad} + t_{div} \tag{31}$$

Combining (28)–(31), the final expression for the duration of the tone detection process is obtained:

$$t_{fx} = T_f \,.\, N + 2 \,.\, L \,.\, (t_{mul} + t_{ad}) + 2 \,.\, (t_{div} - t_{ad}) + L \,.\, (t_{ph} + \log_2 N \,.\, t_{sh}) \tag{32}$$

where T_f is the period of the expected frequency; t_{sh} is the time for 'logical shift'; t_{mul} is the time for multiplication; t_{div} is the time for division; t_{ad} is the time for addition; and t_{ph} is the time for phase comparison.

Based on the A/D conversion time, t_{adc}, the number of samples per period, must satisfy the following inequality:

$$L \leq T_f / t_{adc} \tag{33}$$

The Table shows the tone detection timing of the implemented controller for the various coding formats. In each entry, the number of samples per period, the number of sampled periods, the data acquisition time and the total execution time when the expected frequency has been detected are shown. The total processing time varies from 7·45 ms for 1446 Hz (EIA character '6', $N=8$, $L=32$), up to 29·91 ms for 1124 Hz (CCIR character '1', $N=32$, $L=20$). It must be emphasized that the microcontroller uses its serial interface to receive commands for the coding format which must be used, the specific pattern which must be detected, and also to communicate with the application processor.

5. Conclusions

A computationally efficient method for tone detection, and its implementation, have been presented. The method is based on the adaptation of the sampling process to the expected frequency value and on the averaging of the generated samples using the same phase criterion. The method is very simple, requires only a small amount of computation and uses simple functions by selecting the implementation parameter appropriately. The main advantage of the method, besides its simplicity and efficiency, is that it can be adapted to the application requirements during system operation and can support various coding formats without requiring any hardware modification.

As the mathematical analysis shows, the method can be used even for very noisy conditions, for various sequential tone signalling systems and, as has been shown, it can be implemented by using a conventional microcontroller. The detailed description of an implementation has been given, emphasizing its ability to implement the various coding standards effectively.

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