# A Bandwidth Allocation Algorithm for Multiuser OFDM Systems and its Efficient Implementation

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***Abstract*— This paper addresses the problem of efficient resource allocation for multiuser orthogonal frequency division multiplexing over frequency selective channels. For wideband applications, such as power line communications and wireless networks, the development of low-complexity and fast execution time algorithms is important due to the time-varying behavior of the channel environment and the need to adapt the bandwidth allocation to the channel conditions. This paper presents a multiuser bandwidth allocation algorithm, examines its complexity and presents a computationally efficient implementation. Numerical results demonstrate the improvement achieved with the proposed implementation, in terms of complexity.**

## I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) is widely used in wireless and wireline communications systems due its capability in combating channel impairments, such as intersymbol interference and frequency selective fading, while achieving high capacity and efficient spectrum utilization [1]. In particular, OFDM is adopted in various wireless applications between a base station and multiple user terminals, such as wireless local area networks (IEEE 802.11) [2] and metropolitan area networks (IEEE 802.16) [3]. Recently, great attention has been devoted to OFDM for power line communications (PLC) as a potential solution for in-home networking [4]. In the indoor PLC network, communication is established between user devices connected at the various termination points of the power grid and a central unit, which is also the interface to the backbone network.

In multiuser OFDM systems, a fundamental issue is the allocation of the available bandwidth and power resources to the different users. Generally, resource allocation is treated independently in the two directions of transmission, namely the downlink (from the central station to the user terminals) and uplink (from the user terminals to the central station). There exist two classes of resource allocation strategies: fixed and dynamic. Fixed resource allocation schemes, such as time-division multiplexing (TDMA) or frequency-division multiplexing (FDMA), assign an independent dimension, e.g. time slot or subchannel, to each user. However, in a frequency selective environment, the various users may experience different channel gains, even in the same frequency bands. Therefore, a fixed resource allocation strategy may result in poor system performance, since it does not utilize the current channel conditions. On the other hand, dynamic resource allocation assigns subchannels adaptively to the various users according to their channel gains. In addition, for time-varying channel environments, dynamic resource allocation exploits the multiuser diversity in order to achieve higher network performance. As the demand for high rate and high performance services is increasing, the development of efficient low-complexity resource allocation algorithms draws particular interest [5], [6].

In general, resource allocation is treated as a constrained optimization problem. The objective function to be maximized (minimized) is usually the total network rate (power), subject to a set of constraints that define system specifications and/or practical limitations, e.g. total power budget, integer number of bits in each subchannel, etc. In addition, a low-complexity solution is of particular interest for applications with time-varying channel environments in order to account for the real-time adaption of the resource allocation to the channel conditions.

In this paper, we address the rate-maximization problem for downlink OFDM systems over frequency selective channels with power-spectral density (PSD) limitations. In order to exploit multiuser diversity, we employ the FDMA multiple access rule, i.e. each subchannel is assigned to one user only, and in order to enhance spectrum utilization, we request that the allocated user data rates are greater than the rates achieved by fixed (equal time-slots) TDMA. We present a multiuser loading algorithm based on a low-complexity round robin allocation scheme with user priorities and we investigate its efficient implementation.

The rest of this paper is organized as follows. Section II formulates the resource allocation problem and Section III describes the multiuser loading algorithm. In Section IV, we present performance results using an indoor PLC network, while in Section V, we examine the computational complexity of the multiuser loading algorithm, we present a low complexity solution, and we provide numerical results that demonstrate the improvement achieved with the proposed implementation in terms of complexity.

## II. THE RESOURCE ALLOCATION PROBLEM

Let $K$ be the number of users and $N$ be the number of subchannels. For each user $k$, we denote as $g_{k,n}$ the channel gain-to-noise ratio of subchannel $n$. The rate in each subchannel is $r_{k,n} = f(p_{k,n} \cdot g_{k,n})$, where $r_{k,n}$ is the number of bits, $p_{k,n}$ is the allocated power, and $f$ is a concave and increasing function that depends on the target error probability, as well as on the applied modulation and coding schemes. Given a PSD constraint, we denote as $\bar{p}$ the maximum power that is allowed in each subchannel. As a result, $r_{k,n}$ is upper-bounded by an integer number of bits calculated by $\bar{r}_{k,n} = \lfloor f(\bar{p} \cdot g_{k,n}) \rfloor$, where $\lfloor \cdot \rfloor$ is the floor function.

Assuming that the central station is able to transmit at the PSD level in all subchannels in order to increase the total network rate (i.e. each user is able to transmit $\bar{r}_{k,n}$ bits in each subchannel), the resource allocation problem turns to a subchannel assignment problem. Since each subchannel can be assigned to one user only, we define the subchannel allocation matrix $\mathbf{A} = [a_{k,n}]_{K \times N}$, where $a_{k,n} = 1$, if subchannel $n$ is allocated to user $k$, otherwise $a_{k,n} = 0$, and the total rate of user $k$ is calculated by $R_k = \sum_{n=1}^{N} a_{k,n} \cdot \bar{r}_{k,n}$. The resource allocation problem is formulated as follows:

$$\begin{aligned} \underset{a_{k,n}}{\text{maximize}} \quad & \sum_{k=1}^{K} \sum_{n=1}^{N} a_{k,n} \cdot \bar{r}_{k,n} & (1) \\ \text{subject to} \quad & \text{(C1)} \sum_{k=1}^{K} a_{k,n} = 1 \qquad \forall n = 1, \ldots, N \\ & \text{(C2)} \sum_{n=1}^{N} a_{k,n} \cdot \bar{r}_{k,n} \geq R_k^{\min} \quad \forall k = 1, \ldots, K \end{aligned}$$

In (1), the optimization objective is the maximization of the total network rate. The first constraint (C1) imposes the FDMA rule, while the second constraint (C2) defines the requirement that a minimum user data rate $R_k^{\min}$ is achieved. In general, the values of $R_k^{\min}$ can be service-oriented. In this paper, we choose to set $R_k^{\min} = \frac{1}{K} \sum_{n=1}^{N} \bar{r}_{k,n}$ for each user. Note, that $\sum_{n=1}^{N} \bar{r}_{k,n}$ is the total rate achieved in the single-user scenario, i.e. when user $k$ utilizes all the available subchannels, and $\frac{1}{K} \sum_{n=1}^{N} \bar{r}_{k,n}$ equals to the user rate achieved in fixed TDMA with equal time-slots for all users. Thus, constraint (C2) implies that the FDMA solution of (1) should outperform the fixed TDMA resource allocation.

Without using constraint (C2), the optimal solution to (1) is determined by assigning each subchannel to the user with the maximum $g_{k,n}$ (and as a result with the maximum $\bar{r}_{k,n}$) [6]. However, this strategy may penalize the users with poor or even moderate channel conditions and thus is does not guarantee any minimum data rates. In the next section we present a multiuser loading algorithm based on round-robin allocation scheme with user priorities [7].

## III. The Multiuser Loading Algorithm

The multiuser loading algorithm is based on an iterative process of allocation rounds. In each round, the algorithm assigns subchannel, bit, and power to all users according to a user-priority order. We define a binary array $\mathbf{U} = [u_{i,j}]_{K \times K}$ that describes the priority of the users in each allocation round: $u_{i,j} = 1$ indicates that user $j$ has $i$-th priority, where $i = 1$ is the highest priority. Clearly, $\mathbf{U}$ contains at most one nonzero element in each column. The algorithm assigns higher priority to the users with the lowest achieved data rate.

More specifically, at the beginning of each allocation round, the users are ordered in ascending order according to their $R_k$ values, and a new user-priority array $\mathbf{U}$ is generated. The result of this process is that several groups of users, having the same $R_k$, may be formed. For a single-user group, we assign the subchannel with the maximum supported rate to that particular user. For a multi-user group, we assign the best subchannels (one for each user) that support the maximum rates.

In order to describe the algorithm, we introduce the following arrays: $\mathbf{G} = [g_{k,n}]_{K \times N}$, $\mathbf{P_{max}} = [\bar{p}_{k,n}]_{K \times N}$, $\mathbf{R_{max}} = [\bar{r}_{k,n}]_{K \times N}$, $\mathbf{R} = [r_{k,n}]_{K \times N}$, $\mathbf{P} = [p_{k,n}]_{K \times N}$. We also denote as $\Omega \subseteq \{1, 2, \ldots, N\}$ the set of available subchannels and as $S_i := \{k = 1, \ldots, K : u_{i,k} \neq 0\}$ the group of users with $i$-th priority in $\mathbf{U}$. The multiuser loading algorithm, named *beaf* as explained in [7], is described as follows.

`Initialization`

- I1. Initialize $\mathbf{A}$, $\mathbf{P}$ and $\mathbf{R}$ with zeros $\forall k, n$.
- I2. Calculate $\mathbf{R_{max}}$ based on $\mathbf{G}$ and $\mathbf{P_{max}}$.
- I3. Set $R_k = 0$ for $k = 1, \ldots, K$ and $\Omega = \{1, 2, \ldots, N\}$.

`Main Resource Allocation`

- A1. Calculate the user-priority array $\mathbf{U}$.
- A2. Perform a best effort allocation round.
  - R1. Start with the group of users with highest priority, $i = 1$.
  - R2. For the users $\{k_1, \ldots, k_K\} \in S_i$ find subchannel $n^* = \arg\max g_{k,n}$ within the rows $k_1, \ldots, k_K$ of $\mathbf{G}$. Let $n^*$ belong to row $k^*$.
  - R3. Assign subchannel $n^*$ to user $k^*$, i.e. $a_{k^*,n^*} = 1$, $S_i = S_i - \{k^*\}$, and $\Omega = \Omega - \{n^*\}$.
  - R4. Update $\mathbf{R}$ and $\mathbf{P}$: $r_{k^*,n^*} = \bar{r}_{k^*,n^*}$ and $p_{k^*,n^*} = f^{-1}(\bar{r}_{k^*,n^*}) \cdot g_{k^*,n^*}^{-1}$.
  - R5. Update the rate of user $k^*$: $R_{k^*} = R_{k^*} + r_{k^*,n^*}$.
  - R6. Repeat steps R1-R5 for the remaining users in $S_i$.
  - R7. Repeat steps R1-R6 with the remaining groups in $\mathbf{U}$, $i = 2, \ldots, K$.
- A3. Repeat steps A1-A2 until $\Omega = \emptyset$.

The resource allocation process is resolved using an outer loop of main allocation rounds. The number of iterations is limited due to the limited resources. Step A.1 provides a fair allocation order, so that the least satisfied users are the first to be assigned resources in each allocation round. At initialization, all users belong to the same group.

## IV. Performance Analysis

In this section we present performance results of the multiuser loading algorithm using the indoor PLC networking environment. In general, the PLC channel is a multipath environment. The channel gains between any two access points depend on the topology, as well as on the network's loading conditions, i.e. the impedances of the various loads that are connected to the power grid. In order to evaluate the algorithm under different channel conditions, we use the 5-user

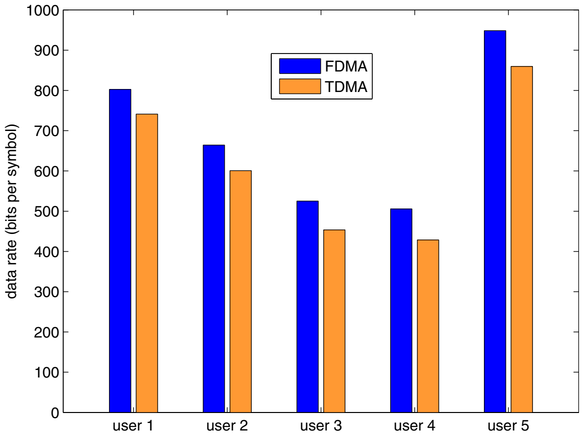


Fig. 1. Average user data rate using FDMA and TDMA strategies.

PLC network with the OFDM communications parameters described in [7].

Fig. 1 shows the average user data rate achieved with the multiuser loading algorithm in Section III compared with the fixed TDMA scheme. We observe that FDMA outperforms TDMA since the former exploits the multiuser diversity in order to increase the total network rate. Indeed, the FDMA strategy allocates each subchannel to the user who is able to utilize the selected subchannel according to a predefined criterion (e.g. maximum bit-rate in the algorithm of Section III). On the contrary, in the TDMA scheme, the subchannels with very low gain-to-noise ratio are wasted during each user's time-slot.

Table I presents the average fairness ratio of each user defined as $\rho_k = R_k / \sum_{n=1}^{N} \bar{r}_{k,n}$. Results are shown for three subchannel allocation strategies:

- *max-sum*: each subchannel is allocated to the user with the maximum gain-to-noise ratio [6].
- *beaf*: corresponds to the best effort allocation scheme in Section III.
- *prop-fair*: at each allocation step, the user with the less proportional rate, $\rho_k$, selects the best of the available subchannels [5].

We observe that the *beaf* loading algorithm provides proportional rate allocation with fairness results similar to the *prof-fair* allocation method. On the other hand, the *max-sum* strategy allocates most of the network resources to user 5 and completely penalizes users 3 and 4. Table 1 also includes the mean and standard deviation (STD) of the users' fairness ratios for each strategy.

## V. Complexity Analysis

In order to investigate the computational complexity of the multiuser loading algorithm, we concentrate on the number of comparisons required at each allocation round. Assume that a user group $S_i$ contains $M$ users. Step R2 finds the $M$ maximum values in a $M \times N_{av}$ sub-array of $\mathbf{G}$, where $N_{av}$ are the available subchannels. After the selection of each maximum value the corresponding row and column are not

TABLE I

Average User Fairness Ratio Values

| User ID | max-sum | beaf | prop-fair |
|---|---|---|---|
| 1 | 0.2024 | 0.2167 | 0.2228 |
| 2 | 0.0226 | 0.2214 | 0.2224 |
| 3 | 0.0000 | 0.2320 | 0.2210 |
| 4 | 0.0003 | 0.2359 | 0.2214 |
| 5 | 0.8377 | 0.2207 | 0.2229 |
| MEAN | 0.2126 | 0.2253 | 0.2221 |
| STD | 0.3596 | 0.0082 | 0.0008 |

encountered until the next allocation round. The number of comparisons is:

$$C = \sum_{i=1}^{\min(M,N_{av})} \left[ (M - i + 1)\,(N_{av} - i + 1) - 1 \right] \quad (2)$$

An analytical expression for the total number of comparisons required for a complete allocation round, steps R.1-R.7, can not be derived, since the complexity depends on the arrangements and the order of the user groups according to the $R_k$ values. In general, $K$ users form $Q(K)$ possible arrangements, where $Q(1) = 1$, $Q(2) = 2$, $Q(3) = 3$, and

$$Q(K) = 3 + \sum_{i=2}^{K-2} Q(K - i), \quad K \geq 4 \quad (3)$$

Fig. 2 shows an example of 3 users. There are 3 arrangements and 4 orders (a)-(d). For $N \geq 3$, the total number of comparisons for each arrangement is given in Table II. It can be shown, that for arbitrary $K$, there exists a minimum complexity arrangement such as case (a) and a maximum complexity arrangement such as case (d). Also, the order of the possible arrangement results in different computational complexities, e.g. case (b) is less complex than case (c).

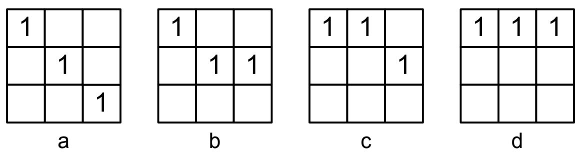


Fig. 2. Groups and orders in a 3 users example.

TABLE II

Computational Complexity for the Example of Fig. 2

| order | comparisons | order | comparisons |
|---|---|---|---|
| a | $3N - 6$ | c | $4N - 6$ |
| b | $4N - 7$ | d | $6N - 7$ |

### A. The Initial Sorting Approach

In order to reduce the complexity of the proposed algorithm, we introduce the 'Initial Sorting Approach', where the rows of **G** are sorted at the initialization phase. Then, step R2 finds the $M$ maximum values among only $M$ elements in the sorted **G**. This has the same complexity as the sorting of $M$ elements. The concept is illustrated in the example of Fig. 3, where we consider 3 users, named $x, y, z$ and 5 subchannels. The lower the subscript, the higher is the element's value within the row. We want to find the 3 maximum values in the $3 \times 5$ array. Case (a) represents the comparison steps for the algorithm in Section III. In (a.1), $x_1$ is selected initially. The row and column of $x_1$ are disabled. In (a.2), $y_1$ is selected and the row and column of $y_1$ are also disabled. Finally in (a.3), $z_2$ is selected. In case (b) the $3 \times 5$ array is sorted per row. In (b.1), $x_1$ is initially selected in the first column. The row of $x_1$, as well as elements $y_2$ and $z_5$ are disabled. In (b.2), $y_1$ is selected and the row of $y_1$ as well as elements $x_3$ and $z_1$ are disabled. In (b.3), $z_2$ is selected and the elements $x_5$ and $y_3$ are disabled.

The number of comparisons for efficiently sorting a vector of $M$ elements is $S_v(M) = M \log M$. Therefore array **G** requires $KS_v(N)$ total comparisons, while step R2 requires $S_v(M)$ comparisons for each group of $M$ users. Note, that for a user arrangement such as Fig. 2(a), no comparisons are required, since each user is allocated the next available of his subchannels. Sorting also requires an additional array for storing the indexes of the original unsorted elements.

### B. Numerical Results

We compare the complexity between the two solutions, i.e. with and without initial sorting of **G**. Since, the total complexity of the *beaf* algorithm depends on the computational load of each allocation round and the total number of rounds, we need to define a reasonable configuration for comparison. For different values of subchannels $N$ and users $K$, we estimate the number of allocation rounds by $\lceil \frac{N}{K} \rceil$. Then, we consider a configuration with minimum complexity arrangements such as (a) in Fig. 2 and one with maximum complexity arrangement such as (d). When **G** is sorted, arrangement (a) requires zero comparisons, while (d) requires $S_v(K)$. On the other hand, when **G** is unsorted, arrangement (a) requires $NK - 0.5K(K+1)$ comparisons, while for (d) the number of comparisons is calculated by (2).

Fig. 4 shows the *complexity improvement factor* (CIF) defined as the ratio between the total number of comparisons of the unsorted and the sorted approach. The results correspond to the mean value of the CIF, where the minimum and maximum values were calculated using the marginal cases of Fig. 2, i.e. case (a) and (d) respectively. The results include the initial overhead for sorting array **G**. We observe that initial sorting provides significant improvement. In general, CIF increases as the number of subchannel increases. For wideband applications, where the number of subchannels is high, the proposed implementation approach provides significant complexity improvement.

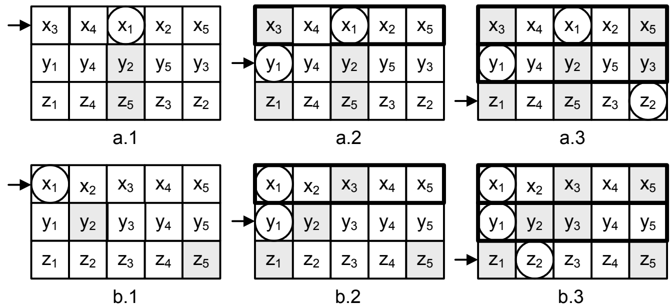


Fig. 3. Allocation steps in the '3 users, 5 subchannels' example. A circle indicates element selection in each round. A bold-outlined row indicates resources assigned to a user in the previous round. Gray shade indicates subchannels allocated to another user.

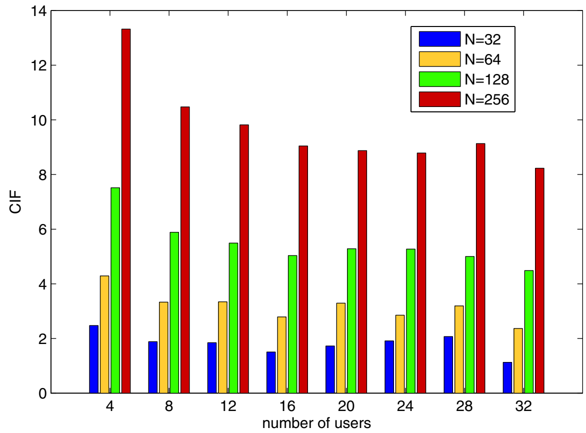


Fig. 4. Complexity comparison between unsorted and sorted approach.

## VI. Conclusion

We presented a multiuser loading algorithm for OFDM systems in frequency selective channels. We examined the algorithm's complexity and we investigated the computational load of possible implementation solutions. Finally, we showed that the initial sorting of the decision's variable array provides significant complexity improvement.

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